Final Exam – MCT 4C

Mr. G Kempe

Friday, June 20th

Name : _____

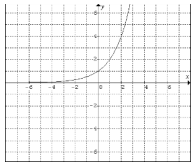
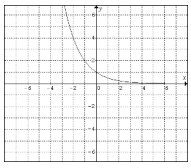
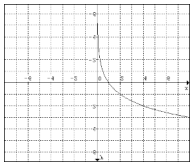
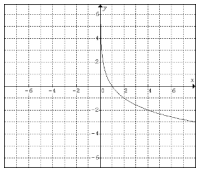
Instructions: There are ***x*** pages in this exam, including this title page.
Answer all questions in the space provided.
There is to be no talking during this exam.
Materials allowed are: pencil, pen, calculator.
Write your name at the top of **every page** of this exam.
There are three sections to this exam:

Part A: Multiple Choice	/ 30
Part B: Definitions / Short Answer	/ 20
Part C: Problem Solving * Complete 10 of 12 questions*	/ 50
Total	**/ 100**

Part A: Multiple Choice

[30 marks]

1. Which of the following shows the graph of $y = \left(\frac{1}{2}\right)^x$?

a.  b.  c.  d. 

2. The domain of $y = \log(x)$ is…

a. $\{x \in R\}$ b. $\{x > 0, x \in R\}$ c. $\{x < 0, x \in R\}$ d. $\{x \neq 0, x \in R\}$

3. When simplified, the expression $(3x^3y^2) \div (6x^4y)$ becomes…

a. $0.5xy$ b. $\frac{2y}{x}$ c. $\frac{2x}{y}$ d. $18x^7y^3$

4. How many times more intense is an "8" earthquake (on the Ritcher scale) than a "4"?

a. 2 b. 4 c. 40 d. 10000

5. If $2^{x-1} = 16$, then $x = \ldots$

a. 4 b. 5 c. 8 d. 9

6. Which of the following equations shows a half-life of 15 minutes?

a. $y = 15(0.5)^x$ b. $y = 10(0.5)^{t/15}$ c. $y = 10(0.5)^{15/t}$ d. $y = 10(15)^{1/2t}$

7. If $\log_3(x-2) = 3$, then $x = \ldots$

a. 5 b. 29 c. 243 d. No solution

8. What is the degree of $y = 4x^2 - 5x^3 + 8$?

a. 2 b. 3 c. 5 d. 8

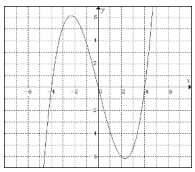
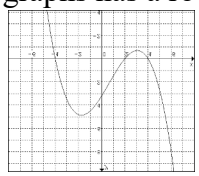
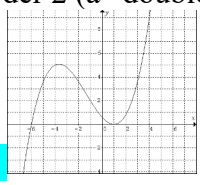
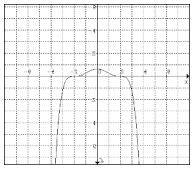
9. What is the degree of $y = -2(x+1)^2(x-3)(x-4)$?

a. –4 b. –2 c. 2 d. 4

10. Which quadrants does $y = -x^5$ enter from / exit to?

a. III and I b. II and IV c. III and IV d. XI and LXVIII

11. Which of the follow graphs has a root of order 2 (a "double" root)?

a.  b.  c.  d. 

12. The 3rd differences of a table of differences are all 12. What does this say about the function?
a. Degree 3; Leading coefficient is 12 b. Degree 3; Leading coefficient is 2
c. Degree 12; Leading coefficient is 3 d. Degree 12; Leading coefficient is 0.5

13. What type of factoring should you always try first?
a. Common Factoring b. Factoring by Grouping
c. Advanced Trinomial Factoring d. Fear Factoring

14. What is the remainder when $30x^3 - 28x^2 + 15x - 2$ is divided by $x - 1$?
a. -75 b. -2 c. 0 d. 15

15. Which of the following binomials is definitely **not** a factor of $2x^4 - ax^3 + bx - 8$?
a. $(x - 1)$ b. $(x - 2)$ c. $(x - 3)$ d. $(x - 4)$

16. An angle in standard position passes through the point (5, 6). Which of the following is true?
a. $\sin\theta = \frac{5}{6}$ b. $\cos\theta = \frac{5}{6}$ c. $\tan\theta = \frac{5}{6}$ d. None of the above

17. Which of the following is a correct statement of the Cosine Law?
a. $a^2 = b^2 + c^2 + 2bc\cos A$ b. $a^2 = b^2 + c^2 - 2bc\cos B$
c. $c^2 = a^2 + b^2 + 2bc\cos C$ d. $b^2 = a^2 + c^2 - 2ac\cos B$

18. Which of the following is equivalent to 245°?
a. S65°E b. S65°W c. S25°E d. S25°W

19. What is the vertical component the following vector: 35 N at 55° to the horizontal.
a. 20N b. 29N c. 25N d. 35N

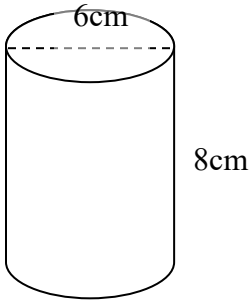
20. A man walks 5 km into a forest bearing 314°. In what direction must he walk to return the way he came?
a. N46°W b. N44°W c. S46°E d. S44°E

21. Which of the following functions has a phase shift of 2 units to the right?
a. $y = 2\sin(x)$ b. $y = \sin(2x)$ c. $y = \sin(x-2)$ d. $y = \sin(x)+2$

22. Which of the following functions has a period of 180°?
a. $y = 2\cos(x)$ b. $y = \cos(2x)$ c. $y = 180\cos(x)$ d. $y = \cos(180x)$

23. Which of the following functions has the highest maximum?
a. $y = 3\cos(x)$ b. $y = 2\sin(x)+6$ c. $y = 5\cos(x)-2$ d. $y = 0.5\cos(x)+8$

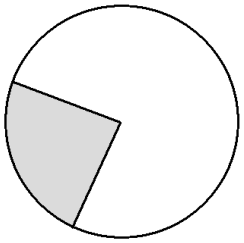
Questions 24 and 25 deal with the cylinder, right.



24. What is the surface area of the cylinder?
a. 207.35cm² b. 251.33cm² c. 527.79cm² d. 703.72cm²

25. What is the volume of the cylinder?
a. 226.19cm³ b. 452.39cm³ c. 904.78cm³ d. 1206.37cm³

26. What is the name for the shaded area in the circle, shown right?
a. Segment b. Sector
c. Chord d. Pac-Man



27. What is the name for the line segment that forms the boundary of the shaded area?
a. Chord b. Arc
c. Radius d. Tangent

28. How many tangent lines can be drawn to a circle from a point outside the circle?
a. 2 b. 1 c. 0 d. –1

29. If two inscribed angles share the same chord, then …
a. They are equal b. One is double the other
c. At least one is 90° d. None of the above

30. Which of the following are perpendicular in/on a circle?
a. Tangent & radius to the point of tangency b. Any two chords
c. Two sides of any inscribed triangle d. All of the above

Bonus.

Part B: Definition / Short Answer

[20 marks]

1. State two properties of the graph of the basic exponential function ($y = b^x$). [2]

   Any two of:
   y-intercept is 1
   Asymptote is $x = 0$
   Increases when $b > 1$; decreases for $0 < b < 1$
   Domain is $\{x \in R\}$; Range is $\{y > 0, y \in R\}$

2. State two properties of the graph of the basic logarithmic function ($y = b^x$). [2]

   Any two of:
   x-intercept is 1
   Asymptote is $y = 0$
   Increases when $b > 1$; decreases for $0 < b < 1$
   Domain is $\{x > 0, x \in R\}$; Range is $\{y \in R\}$

3. When asked to evaluate $\log_2 32$, what are you being asked to do? [1]

   Determine the exponent, *x*, such that $2^x = 32$.

4. Explain how you can use the degree of a function and its leading coefficient to determine its end behaviour. [2 marks]

	Odd Degree	Even Degree
Positive "a"	Q3 to Q1	Q2 to Q1
Negative "a"	Q2 to Q4	Q3 to Q4

5. You are given a table of values for a polynomial function. How can you determine the degree of the function? [1 mark]

   Take finite differences until the values are constant. When you reach a column with a constant difference, the number of columns is the degree. (i.e. 3rd differences are constant means a 3rd degree function.)

6. Explain how to factor a polynomial using the "guess and divide" method. [2 marks]

   Test values of x until you get a result of 0. Divide the corresponding factor.

7. Define *parallel*, *equivalent*, and *opposite* vectors. [3 marks]

   Parallel – same or opposite directions
   Equivalent – same magnitude and direction
   Opposite – same magnitude, opposite directions

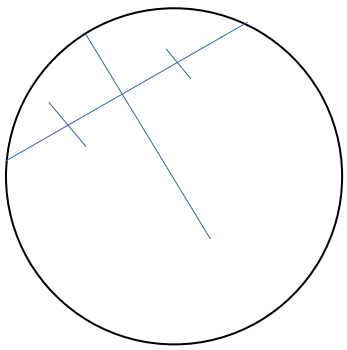
8. State all transformations in the function $y=-2\cos\left[4\left(x+30\right)\right]+2$. [3 marks]

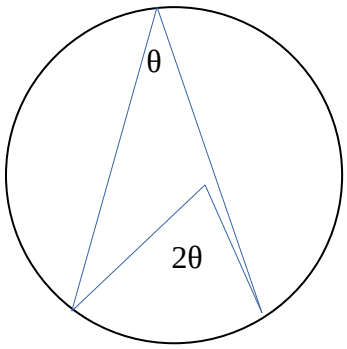
   Amplitude (vertical stretch) is 2. Vertical reflection.
   Horizontal compression by ¼. Period is 90.
   Phase shift of 30 left.
   Vertical shift up 2 (center line).

9. There are 12 inches in one foot. Are there 12 cubic inches in one cubic foot? Explain. [2 marks]

   No. There are $12^3 = 1728$ cubic inches in one cubic foot.

10. Draw the following circle properties: [2 marks]
    a. The perpendicular bisector of a chord passes through the centre of a circle.
    b. A central angle is twice the size of an inscribed angle that shares the same chord.





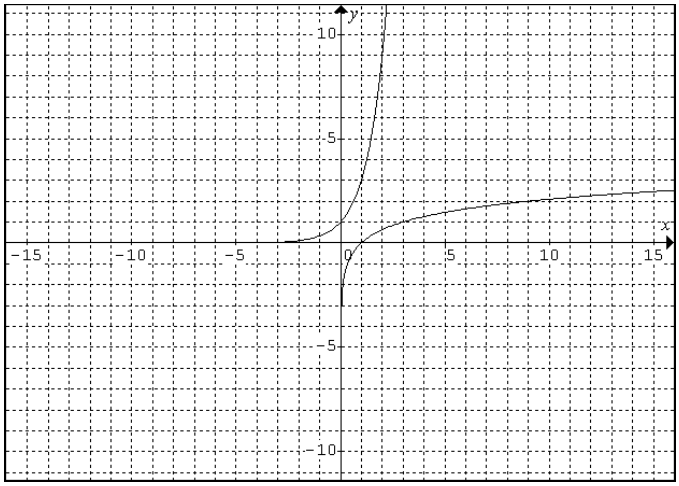
Part C: Problem Solving

[50 marks]

1. Sketch the graphs of $y=3^x$ and $y=\log_3 x$ on the same axes. [5]

x	3^x
-3	1/27
-2	1/9
-1	1/3
0	1
1	3
2	9
3	27

x	$\log_3 x$
1/27	-3
1/9	-2
1/3	-1
1	0
3	1
9	2
27	3



2. Lake Chad, in Eastern Africa, covered 25000km² in 1983. The table below shows its size over subsequent years. Assuming this situation can be modelled by exponential decay,

Year	Size
1983	25000
1984	21750
1985	18900
1986	16450
1987	14300

   a. Determine the decay rate of the lake. [3]

   $21750=25000(b)^1$
   $0.87=b$

   1 – 0.87 = 0.13

   The lake loses about 13% each year.

   b. Predict the current size of the lake (2014). [2]

   $y=25000(0.87)^{31}$
   $y=333.5$

   About 330km²

3. Angela invests $1500 into a GIC earning 3% annually. Serene invests $1000 into a mutual fund earning 5.2% annually. Which investment will reach $2000 first? [5]

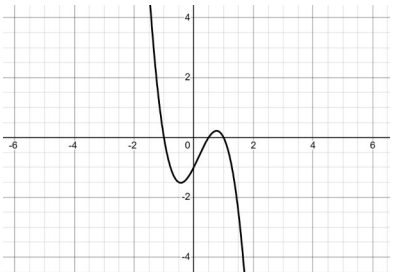
$2000 = 1500(1.03)^x$
$1.3333 = 1.03^x$
$\log 1.3333 = x \log 1.03$
$9.73 = x$

$2000 = 1000(1.052)^x$
$2 = 1.052^x$
$\log 2 = x \log 1.052$
$13.67 = x$

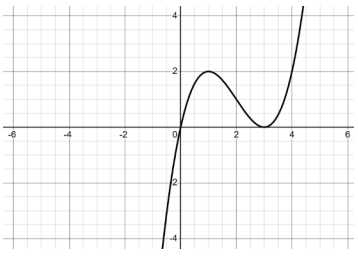
Angela's.

4. Sketch the graph of the following functions *without* using graphing technology. Make your graph as accurate as possible. Please show any rough work in the space below the graph. [5 marks]

a) $y = -(x-1)(x+1)(2x-1)$



b) $y = 0.5x(x-3)^2$



5. Factor each expression. [5 marks]

a. $4a^3-20a^2+25a-125$

$=4a^2(a-5)-25(a-5)$
$=(a-5)(4a^2-25)$
$=(a-5)(2a-5)(2a+5)$

b. $7x^3-14x^2+7x$

$=7x(x^2-2x+1)$
$=7x(x-1)(x-1)$
$=7x(x-1)^2$

6. The concentration of the painkiller Darnitol™ in the bloodstream is given by the function $C=-0.005t^4+0.15t^3-1.5t^2+5t$ where t is the time, in hours, since the pill was ingested, and C is the concentration in parts per million (ppm)
   a. Use technology to graph this function. Draw a rough sketch of the shape of the graph in the box. [1 mark]

   b. Based on the graph, describe what happens to Darnitol over time. [2 mark]

   c. Darnitol™ is effective (i.e. gives relief) when the concentration is above 1ppm. How long does it take before it becomes effective? How long does the relief last? [2 marks]

7. In triangle ABC, a = 10cm, b = 18cm, and angle A is 30°. Determine all possible measures of the remaining sides and angles. [5 marks]

$h = b\sin A = 18\sin 30 = 9$ The height is less than a, so there are two possible triangles.

$$\frac{\sin B}{18} = \frac{\sin 30}{10}$$
$$\sin B = 0.9$$

B = 64.2 or B = 180 – 64.2 = 115.8

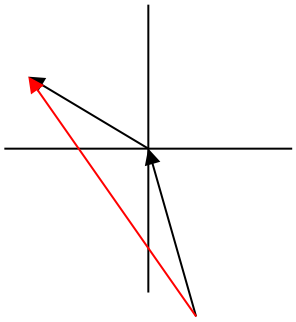
C = 180 – 30 – 64.2 = 85.8 or C = 180 – 30 – 115.8= 34.2

$$\frac{c}{\sin 85.8} = \frac{10}{\sin 30} \qquad \frac{c}{\sin 34.2} = \frac{10}{\sin 30}$$
$$c = 19.95 \qquad c = 11.24$$

8. A fishing boat sets out from the coast of Newfoundland at 18 knots on a course of N65°W. Suddenly, the "Storm of the Century" whips up a current of 40 knots from S15°E. At what speed and in what direction is the boat now travelling? [5 marks]

$$c^2 = 18^2 + 40^2 - 2(18)(40)\cos 130$$
$$c = 53.38$$

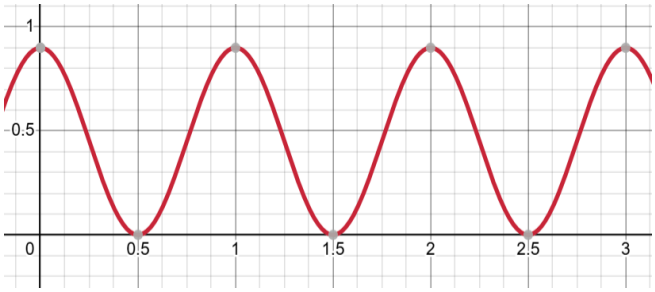
$$\frac{\sin\theta}{18} = \frac{\sin 130}{53.38}$$
$$\theta = 15°$$



Therefore the boat is now travelling at 53.38 knots [N30°W]

9. Angela is dribbling a basketball. The ball's vertical position varies between the floor (0m) and below Angela's hand (0.9m). Each dribble takes 1 second to complete.

a. Sketch the graph of the relationship between the height of the ball and time, starting from Angela's hand. [2 marks]



b. Write an equation that models this relationship. [1 mark]

$y = 0.45\cos(360x) + 0.45$

c. Serene will attempt to steal the ball when the ball rises to 0.7m. At what time will she try to steal the ball? [1 mark]

From the graph, at about 0.85s.

$0.7 = 0.45 \cos(360x) + 0.45$
$0.25 = 0.45 \cos(360x)$
$0.5555 = \cos(360x)$
$56.25 = 360x$ or $360 - 56.25 = 360x$
$0.15265 = x$ or $0.84375 = x$

The second value is when the ball is rising, so at about 0.84s.

10. The "seat" of a Zorb represents 30% of the volume of the full Zorb. If the diameter of the zorb is 3m, what is the diameter of the seat? [5 marks]

$V = \frac{4\pi r^3}{3}$

$V = \frac{4\pi(1.5)^3}{3}$

$V = 14.1$

$4.2 = \frac{4\pi r^3}{3}$

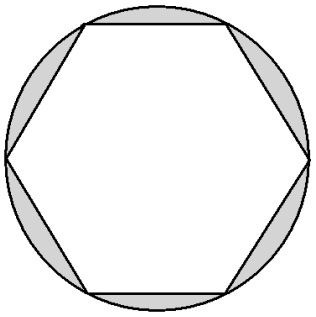
$12.6 = 4\pi r^3$

$1 = r^3$

$1 = r$

The diameter of the seat is 2m.

11. A regular hexagon (just) sits inside a circle with a radius of 12cm. Determine the area of the shaded region. [5 marks]



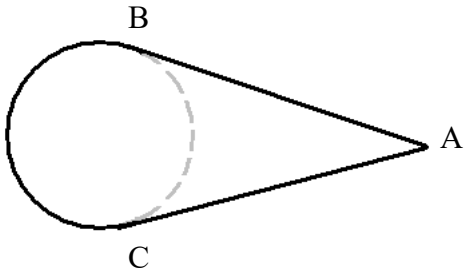
The hexagon is made up of 6 equilateral triangles.

For each sector:

$$A = \frac{60}{360}\pi(12) - \frac{6\left(3\sqrt{3}\right)}{2} = 59.8$$

The total area is 59.6 × 6 = 358.9

12. A jogger starts at point A, jogs 225m and joins a circular track at B. She follows it around to C, then jogs straight back to A. If the radius of the track is 100m, determine the total distance the runner jogged. [5 marks]
The straight parts of the path are given; we need the arc along the circle.



$$\tan O = \frac{225}{100}$$
$$O = 66°$$

Thus angle BOC totals 132°; the *reflex angle* BOC is 228°.

$$\text{arc length} = \frac{228}{360}\left(2\pi(100)\right) = 397.9$$

The total distance run is 2(225) + 397.9 = 847.9m