Exponent Laws

Recall: Exponent Laws

Product Law: $x^a \cdot x^b = x^{a+b}$

Zero Exponent Law: x^0

Quotient Law: $x^a \div x^b = x^{a-b}$

Negative Exponent Law: $x^{-a} = \frac{1}{x^a}$

Power-to-a-Power Law: $\left(x^a\right)^b = x^{ab}$

Example 1: Simplify.

a. $4^3 \times 4^5 = 4^8$

b. $6^5 \div 6^2 = 6^3$

c. $\left(9^5\right)^3 = 9^{15}$

d. $14^0 = 1$

e. $5^{-3} = \frac{1}{5^3}$

Based on the above laws, we see that…

A positive integer exponent means repeated multiplication. [i.e. $6^5 = 6 \times 6 \times 6 \times 6 \times 6$]

A negative integer exponent means repeated division [i.e. $6^{-5} = \frac{1}{6^5}$]

An exponent of zero equals 1. [i.e. $6^0 = 1$]

What about non-integer exponents? What does the expression $36^{\frac{1}{2}}$ mean?

Example 2

a. Evaluate the following using a calculator.

i. $4^{\frac{1}{2}}$	ii. $9^{\frac{1}{2}}$	iii. $16^{\frac{1}{2}}$	iv. $25^{\frac{1}{2}}$
$= 2$	$= 3$	$= 4$	$= 5$

b. What operation is $x^{\frac{1}{2}}$ equivalent to?

Based on the above results, $x^{\frac{1}{2}} = \sqrt{x}$.

c. Evaluate $36^{\frac{1}{2}}$ and $49^{\frac{1}{2}}$ without using a calculator.

$36^{\frac{1}{2}} = \sqrt{36} = 6$; $49^{\frac{1}{2}} = \sqrt{49} = 7$

Rational Exponents

In general, $x^{\frac{1}{n}} = \sqrt[n]{x}$.

Example 3: Rewrite the following in radical form, then evaluate.

a. $64^{1/3}$
$= \sqrt[3]{64}$
$= 8$

b. $1024^{1/5}$
$= \sqrt[5]{1024}$
$= 4$

c. $43046721^{1/16}$
$= \sqrt[16]{40346721}$
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What about sometime like $27^{2/3}$? In this case, we apply the Power-of-a-Power Rule… backwards.

$= 27^{1/3 \times 2} = \left(27^{1/3}\right)^2 = \left(\sqrt[3]{27}\right)^2 = (3)^2 = 9$

In general, $x^{m/n} = \left(\sqrt[n]{x}\right)^m$.

Example 4: Simplify and evaluate: $9^{1/6} \times 9^{1/3}$

$9^{1/6} \times 9^{1/3}$
$= 9^{3/6}$
$= 9^{1/2}$
$= \sqrt{9}$
$= 3$

Practice: pg. 61 # 1abcd, 2ace, 3, 4, 5, 9