

Logarithms

Example 1: What power of 3 is 43046721? Is 56033284?

We are looking for a solution to $3^x = 43046721$. After playing with the calculator for a bit, we find that 16 is the solution.

As for $3^x = 56033284$, that's a bit trickier, as 56033284 is not a "nice" power of 3. We can guess with the calculator, or perhaps you are familiar with the "trick" of

$$\frac{\ln 56033284}{\ln 3} \text{ or } \frac{\log 56033284}{\log 3} = 16.2399935$$

So what are these log and ln things, anyway?

Definition: The **logarithmic function** is the inverse of the exponential function.

$$\text{If } y = \log_a x, \text{ then } x = a^y.$$

(read "y equals the logarithm of x to the base a.")

Example 2: Express each of the following in logarithmic form.

1. $x = 5^y$ ($y = \log_5 x$)
2. $1000 = 10^3$ ($3 = \log_{10} 1000$)
3. $\frac{1}{8} = 2^{-3}$ ($-3 = \log_2 \frac{1}{8}$)

Example 3: Express in exponential form.

1. $\log_{10} 10000 = 4$ ($10000 = 10^4$)
2. $\log_4 \frac{1}{4} = -1$ ($\frac{1}{4} = 4^{-1}$)

Example 4: Evaluate the following logarithms.

1. $\log_2 32$ (5)
2. $\log_7 343$ (3)
3. $\log_3 \frac{1}{27}$ (-3)
4. $\log_x 1$ (0)

Your calculator "automatically" uses base 10. ("The common logarithm.")

$$\text{i.e. } \log 0.001 = -3$$

Practice: pg. 75 #1ace, 3ace, 4ace, 5, 7, 8ace