# Solving Equations using Logarithms

Definition: The Power Law of Logarithms

$$\log_b\left(m^p\right) = p\log_b m$$

Example 1: Rewrite each expression using the power law.

$\log_2\left(x^2\right)$
$= 2\log_2 x$

$\log_7\left(49^x\right)$
$= x\log_7 49$
$= x(2)$
$= 2x$

We use this law when solving equations.

Example 2: Solve for $x$: $2^x = 37$.

(You've probably seen the "trick" using log before; now we explain it a bit!)

Since 37 cannot be easily written as a power of 2, we take the logarithm of both sides so we can "move" the variable out of the exponent.

$\log 2^x = \log 37$
$x\log 2 = \log 37$ (Power Law)
$x = \dfrac{\log 37}{\log 2}$
$x \doteq 5.21$

Example 3: Solve for $x$: $2^{x+2} = 72^3$

$\log 2^{x+2} = \log 72^3$
$(x+2)\log 2 = 3\log 72$
$(x+2) = \dfrac{3\log 72}{\log 2}$
$x = 18.51 - 2$
$x = 16.51$

Example 4: Solve for $x$: $3^{x+4} = 4^x$

This one is a bit different, because we have $x$ on both sides.

$$\log 3^{x+4} = \log 4^x$$
$$(x+4)\log 3 = x\log 4$$
$$x+4 = x\frac{\log 4}{\log 3}$$
$$x+4 = x(1.26)$$
$$4 = 1.26x - x$$
$$4 = 0.26x$$
$$15.38 = x$$