Applications of Exponential Functions Revisited

Example 1: An investment of $1200 earns 5% annually. How long will it take to reach $3000?

$$y = a(b)^x$$
$$3000 = 1200(1.05)^x$$
$$\frac{3000}{1200} = 1.05^x$$
$$2.5 = 1.05^x$$

$$\log 2.5 = x \log 1.05$$
$$x = \frac{\log 2.5}{\log 1.05}$$
$$x = 18.78$$

It will take about 19 years.

Example 2: The population of a small city is tracked over 5 years. The results are shown in the table.

Year	Population
2006	28500
2007	29100
2008	29650
2009	30250
2010	30850

a. Determine a model for the population growth.

To determine the growth rate, we divide consecutive populations, and look for a pattern:

$$\frac{29100}{28500} = 1.021;\ \frac{29650}{29100} = 1.019;\ \frac{30250}{29650} = 1.020;\ \frac{30850}{30250} = 1.020$$

All the numbers are close to 1.02, so we'll use that. If we take 2006 as our starting year,

$$y = 28500(1.02)^x$$

b. Predict the population in 2050.

$$y = 28500(1.02)^{44}$$
$$y = 68117$$

c. Predict the year the population surpasses 100000.

$$100000 = 28500(1.02)^x$$
$$\frac{100000}{28500} = (1.02)^x$$
$$3.5088 = 1.02^x$$
$$\log 3.5088 = x \log 1.02$$
$$63.4 = x$$

2006 + 64 = 2070; by the year 2070, if the trend continues.

Example 3: The half-life of Francium is about 22 minutes. If Mr. Ward has a 1kg sample of Francium on his desk at 8:30, how much is left at 10:30?

Since the length of the half-life is not a "nice" number (1 minute, 1 hour, etc.) we need to convert our amount of time into "half-lives". To do this, we would divide the amount of time by 22, so we reflect that in our exponent.

$y = 1(0.5)^{t/22}$

$y = (0.5)^{120/22}$ There would be 0.0228kg, or 22.8g left.

$y = 0.0228$

Example 4: A car worth $19,000 in 2004 is worth $2,500 today. What is the rate of depreciation?

$y = a(b)^x$

$2500 = 19000(b)^{10}$

$\frac{2500}{19000} = b^{10}$ The rate of depreciation is 1 – 0.82 = 0.18 = 18%.

$0.13178947 = b^{10}$

$\sqrt[10]{0.13178947} = b$

$0.82 \doteq b$

Practice: pg. 70 #8, pg. 73 #11, 12, 13; pg. 78 #7, 8