

## Finite Differences in Polynomial Functions

### Activity: Investigating Finite Differences

<p>Group A: <math>y = x^2</math>  <span style="color: red; font-size: 1.2em;">2nd   2</span></p>	<p><math>y = 2x^3 - 3x + 1</math>  <span style="color: red; font-size: 1.2em;">3rd   12</span></p>	<p style="text-align: center;"><span style="color: red; font-size: 1.2em;"><math>3x + 1</math></span></p> <p><math>y = -0.5x^4 - 2</math>  <span style="color: red; font-size: 1.2em;">1st   3</span>  <span style="color: red; font-size: 1.2em;">4th   -1.2</span></p>
<p>Group B: <math>y = 3x^2 - 5x + 1</math>  <span style="font-size: 1.2em;">2nd   6</span></p>	<p><math>y = -x^3 + 8</math>  <span style="font-size: 1.2em;">3rd   -6</span></p>	<p><math>y = x^4</math>  <span style="font-size: 1.2em;">4th   24</span></p>
<p>Group C: <math>y = -2x^2 + 8</math>  <span style="color: blue; font-size: 1.2em;">2nd   -4</span></p>	<p><math>y = 4x^3</math>  <span style="color: blue; font-size: 1.2em;">3rd   24</span></p>	<p><math>y = 3x^4 + 6x^2 - 18</math>  <span style="color: blue; font-size: 1.2em;">4th   72</span></p>

### Summary of Activity

- An  $n^{\text{th}}$  degree polynomial has an  $n^{\text{th}}$  constant finite difference
- The leading coefficient determines the value of the constant finite difference

### Definition: Factorials

6! = 6 × 5 × 4 × 3 × 2 × 1

The expression  $n!$  means  $n \times (n - 1) \times (n - 2) \times \dots \times 3 \times 2 \times 1$ . It is read as " $n$  factorial".

- The constant finite difference of a polynomial is  $a \times n!$ , where  $a$  is the leading coefficient and  $n$  is the degree.

Example 1: For each function, state which finite difference is constant, and the value of the constant finite difference.

$$f(x) = -4x^3 - 4x + 1$$

3<sup>rd</sup> differences  
are constant

$$a \times n!$$

$$= -4 \times 3!$$

$$= -24$$

$$f(x) = 0.5x^6 + 2x^4$$

6<sup>th</sup> differences

$$0.5 \times 6! = 360$$

Example 2: A polynomial's 4<sup>th</sup> differences are a constant value of 6. State the degree and leading coefficient of the polynomial.

Degree: 4

Coefficient:

$$a \times n! = 6$$

$$a \times 4! = 6$$

$$a \times 24 = 6$$

$$a = \frac{6}{24}$$

$$a = \frac{1}{4}$$

Polynomial  
starts with  
 $\frac{1}{4}x^4 \dots$

Practice: pg. 88 #11, 12, 13, 14