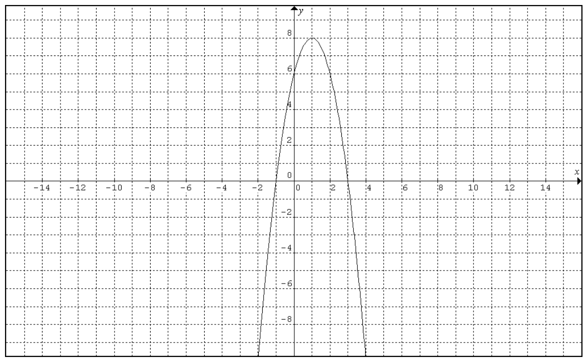
Equations and Graphs of Polynomial Functions

Example 1: Sketch the graph of $f(x) = -2(x+1)(x-3)$.

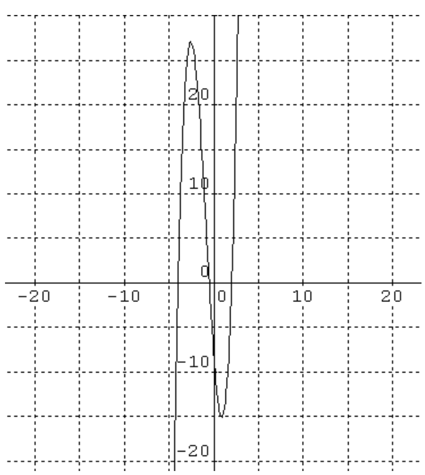
This is a parabola with **zeroes of –1 and 3**, and will **open down**. Its **y-intercept is 6**.



We can graph higher degree functions in the same way, so long as there are factored.

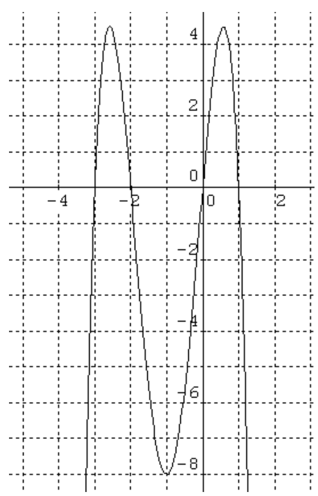
Example 2:
(a) Sketch the graph of the following functions by determining their zeroes, y-intercepts, degree, and end behaviours.

(a) $f(x) = (x-2)(x+4)(2x+1)$



Zeroes: 2, -4, -½
Y-Int: -8
Degree: 3

(b) $f(x) = -2x(x-1)(x+3)(x+2)$



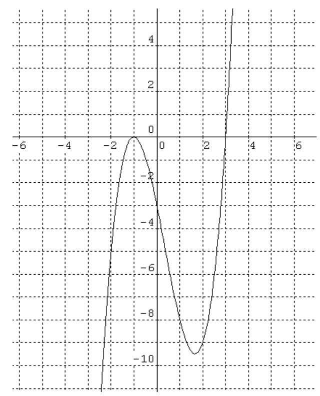
Zeroes: -3, -2, 0, 1
Y-Int: 0
Degree: 4

(b) Identify the intervals of the graph where $f(x)$ is positive and is negative.

(i) $f(x)=(x-2)(x+4)(2x+1)$ is positive on the intervals (-4, -½) and (2, ∞), and negative on the intervals (-∞, -4) and (-½, 2).

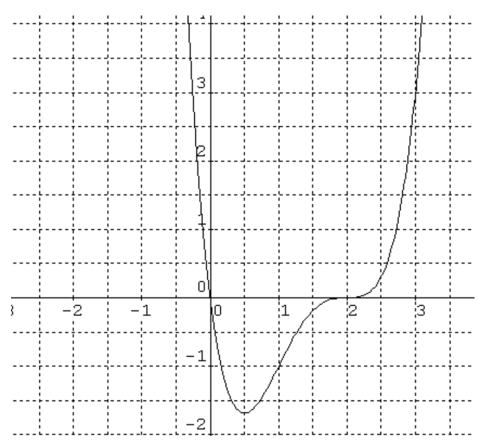
(ii) $f(x)=-2x(x-1)(x+3)(x+2)$ is positive on the intervals (-3, 2) and (0,1) and is negative on the intervals (-∞, -3), (2,0), and (1, ∞).

<u>Example 3:</u> Sketch the graph of $g(x)=(x+1)^2(x-3)$.



Note that there are technically *3* roots here – 3, and –1 twice. (We say the –1 is a root of *order 2*, because there are 2 of them.)
The y-intercept is –3. The degree is 3.

<u>Example 4:</u> Sketch the graph of $f(x)=x(x-2)^3$



Four roots. (Triple root of 2 - *order 3*)
Y-int 0.
Degree 4.

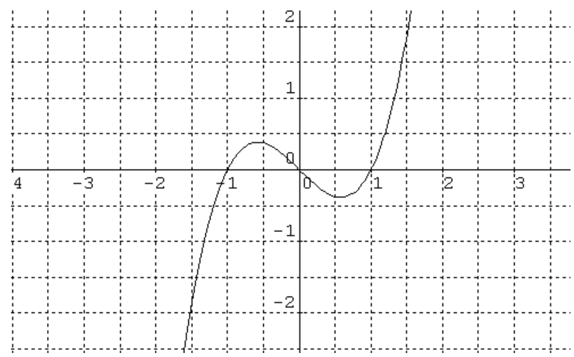
<u>Practice:</u> pg. 108 #8ac, 9ac, 13

Example 5: What kind of symmetry does the function $f(x)=x^3-x$ have, if any?

Option 1: Graph!

$f(x)=x^3-x=x(x^2-1)=x(x+1)(x-1)$

From the graph, the function has odd symmetry.



Option 2: Check if $f(-x)=f(x)$ or $f(-x)=-f(x)$

$$f(-x)=(-x)^3-(-x)$$
$$f(-x)=-x^3+x$$
$$f(-x)=-(x^3-x)$$
$$f(-x)=-f(x)$$

So $f(x)$ has odd symmetry.

Homework: pg. 39 #1, 2, 5ace, 11, 12, 16