

2.4 MULTIPLYING POLYNOMIALS

In Section 2.2 we were able to add and subtract like terms using the distributive property.

$$\begin{aligned} 3x + 7x &= (3 + 7)x \\ &= 10x \end{aligned}$$

In this section we shall use the distributive property to multiply polynomials.

$$\begin{aligned} (3x + 2)(2x - 5) &= (3x + 2)(2x) + (3x + 2)(-5) \\ &= 6x^2 + 4x - 15x - 10 \\ &= 6x^2 - 11x - 10 \end{aligned}$$

To find the product of two polynomials, multiply each term of one of the polynomials by each term of the other and then add the products.

EXAMPLE 1. Expand and simplify. $(2x^2 - x - 1)(x^2 - 2x + 3)$

SOLUTION:

$$\begin{aligned} (2x^2 - x - 1)(x^2 - 2x + 3) &= 2x^4 - 4x^3 + 6x^2 - x^3 + 2x^2 - 3x - x^2 + 2x - 3 \\ &= 2x^4 - 5x^3 + 7x^2 - x - 3 \end{aligned}$$

Squaring a polynomial is simply finding the product of two polynomials.

$$\begin{aligned} (x^2 - 3x + 1)^2 &= (x^2 - 3x + 1)(x^2 - 3x + 1) \\ &= x^4 - 3x^3 + x^2 - 3x^3 + 9x^2 - 3x + x^2 - 3x + 1 \\ &= x^4 - 6x^3 + 11x^2 - 6x + 1 \end{aligned}$$

Three special cases of binomial products are now illustrated.

$$\begin{aligned} (x + 3y)^2 &= (x + 3y)(x + 3y) \\ &= x^2 + 3xy + 3xy + 9y^2 \\ &= x^2 + 6xy + 9y^2 \end{aligned} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} (a + b)^2 = a^2 + 2ab + b^2$$

$$\begin{aligned} (3x - 4)^2 &= (3x - 4)(3x - 4) \\ &= 9x^2 - 12x - 12x + 16 \\ &= 9x^2 - 24x + 16 \end{aligned} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} (a - b)^2 = a^2 - 2ab + b^2$$

$$\begin{aligned} (3x - 2y)(3x + 2y) &= 9x^2 - 6xy + 6xy - 4y^2 \\ &= 9x^2 - 4y^2 \end{aligned} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} (a - b)(a + b) = a^2 - b^2$$

Squaring a trinomial produces a pattern similar to the one resulting when a binomial is squared.

$$\begin{aligned} (2r - 3s + t)^2 &= (2r - 3s + t)(2r - 3s + t) \\ &= 4r^2 - 6rs + 2rt - 6rs + 9s^2 - 3st + 2rt - 3st + t^2 \\ &= 4r^2 + 9s^2 + t^2 - 12rs + 4rt - 6st \end{aligned}$$

$$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc$$

EXAMPLE 2. Expand and simplify.

$$2(2x - 3)(x + 1) - 3(2x - 1)^2$$

SOLUTION:

$$\begin{aligned} 2(2x - 3)(x + 1) - 3(2x - 1)^2 &= 2(2x^2 + 2x - 3x - 3) - 3(4x^2 - 4x + 1) \\ &= 2(2x^2 - x - 3) - 3(4x^2 - 4x + 1) \\ &= 4x^2 - 2x - 6 - 12x^2 + 12x - 3 \end{aligned}$$

EXERCISE 2.4

A 1. Expand.

(a) $3(x + y)$ (b) $x(x + y)$ (c) $3(x - 2)$
(d) $x(x + 7)$ (e) $4x(x - 5)$ (f) $2x(3 - 2x)$

B 2. Expand.

(a) $(x + 3)^2$ (b) $(x - 2)(x + 2)$
(c) $(m - x)^2$ (d) $(r + 7)(r - 7)$
(e) $(x - y)(x + y)$ (f) $(2m + 1)^2$
(g) $(2x - 3y)(2x + 3y)$ (h) $(1 - x)^2$
(i) $(3x - 4y)^2$ (j) $(5s + 3t)^2$
(k) $(2 - 3st)^2$
(l) $(3x^2 - 2y)(3x^2 + 2y)$

3. Expand.

(a) $(x + 3)(x + 2)$ (b) $(y + 4)(y + 5)$
(c) $(y - 3)(y - 7)$ (d) $(t + 3)(t - 4)$
(e) $(x - 7)(x + 3)$ (f) $(m - 5)(m - 7)$
(g) $(t + 5)(t + 11)$ (h) $(x + 10)(x + 11)$
(i) $(x^2 - 3)(x^2 + 6)$ (j) $(x^2 + 1)(x^2 - 2)$
(k) $(x^3 - 8)(x^3 + 6)$ (l) $(1 - x)(5 + x)$
(m) $(10 - x)(8 - x)$ (n) $(7 - t)(8 + t)$

4. Expand and simplify.

(a) $2(x - 4) - 3(x + 2)$
(b) $2(x^2 - 7x + 5) - 3(x - 4)$
(c) $5(3x - 4y) - (2x - 5y) + 7$
(d) $3(r - 2s - t) - 3(4r + 2s - 6t)$
(e) $3(2x - 4) - 3 - (2x + 1) + 5$
(f) $5(3x - 1) - 4(5y + 2) - 6$
(g) $2(2x^2 - 3x + 1) - 4(3x + 5)$
(h) $2x(3x - 5) - 4(2x + 7) + x^2$
(i) $2(1 - 3x + 2x^2) - (1 - 4x + 5x^2)$
(j) $2m(1 - 3m) - m(2m - 3) + m$
(k) $3(x_1 - 2x_2 + 3x_3) - 2(x_2 - x_3)$
(l) $4(2x^2 - 3xy + 4y^2) - 2(x^2 - 3y^2)$

5. Expand and simplify.

(a) $(3x + 4)(x + 5)$ (b) $(2t + 1)(3t + 7)$
(c) $(3x - 4)(2x - 1)$ (d) $(3m - 8)(2m - 3)$
(e) $(4x + 3)(5x - 4)$ (f) $(2r + 7)(3r - 1)$
(g) $(3 - 5y)(1 - 6y)$ (h) $(1 - 3m)(2m + 5)$
(i) $(3x + y)(2x - 3y)$
(j) $(4x - 5y)(3x - 10y)$
(k) $(6w - 11x)(w + 3x)$
(l) $(7x + 2y)(8x - 7y)$
(m) $(5x^2 - 4x)(3x^2 + 2x)$
(n) $(2m - 3m^2)(m^2 + 2m)$

6. Expand and simplify.

(a) $(x + y + z)^2$ (b) $(w - x - y)^2$

(e) $(1 - 3x - 4x^2)^2$ (f) $(5m - 3n + 4)^2$

7. Find the following products.

(a) $(2x + 3)(x^2 + 2x + 1)$
(b) $(3w^2 - 4w - 3)(2w - 1)$
(c) $(2m^2 + 3m - 1)(4m^2 - 2m + 3)$
(d) $(2w - 3x + 2y)(4w - x + 4y)$
(e) $(1 - 3x - x^2)(2 + 4x - 5x^2)$
(f) $(3x - 4y + 2z)(x + 3y - z)$
(g) $(x^3 - x^2 + x - 1)(x^2 - x - 3)$
(h) $(x^3 - x^2 - 2x - 3)(x^3 + 2x^2 + 3x + 1)$
(i) $(m^3 - 2m^2 - 3m - 1)(2m - 5)$
(j) $(3x - 4)(x^3 - 2x^2 + 5x - 4)$

8. Expand and simplify.

(a) $2(x - 4)(x + 3) + 5(2x - 1)(x + 6)$
(b) $3(2t - 5)(t - 4) - 3(5t - 3)(t + 4)$
(c) $2(m - 3)(m - 4) - 3(m + 5)^2 - 2(2m - 1)(2m + 1)$
(d) $3(2m + 3)^2 - (m - 5)^2 - (2m - 4)(m - 5)$
(e) $5(2x - 5)(2x + 5) - 4(x - 2)(x + 3) - (2x + 1)^2$
(f) $(1 - 3x)(2 + 5x) - (x - 4)(2x - 5) - (2x + 3)^2$
(g) $5(2x - 3) - 2(x - 4)(x - 5) + 3x^2 - (x - 6)$
(h) $5x^2 - (x - 3)^2 - 2(x^2 - 5x) + 2(2x - 3)^2$
(i) $1 - (1 - 3x) - (x + 5)^2 - (3 - 4x)^2 + 6x^2$
(j) $(x - y)(x + 2y) - 3(2x - 3y)(x - 4y) + 3(x + y)^2$
(k) $(2w + 3x)(w - x) - 4(w - 2x)^2 + 5(w^2 - x^2)$
(l) $4(x^2 - 3xy) - (x + y)^2 - 2(x - y)(x + y) + 5$
(m) $2(x - 1)(x^2 - 3x + 2) - (2x^2 - 3x - 4)(2x + 3)$
(n) $5(r - s + t)(r - 2s - 3t) - (r + s + t)^2 - (r - s - 3t)$

C 9. Expand and simplify.

(a) $(2x - 1)(x + 4)(3x - 5)$
(b) $(x - 2y)(x + 3y)(2x - 5y)$
(c) $(w + x + y + z)^2$
(d) $\left(x + \frac{1}{x}\right)\left(x - \frac{1}{x}\right)$ (e) $\left(m - \frac{2}{m}\right)\left(m + \frac{3}{m}\right)$
(f) $\left(1 - x + \frac{1}{x}\right)\left(2 + x - \frac{1}{x}\right)$