

Review – Factoring Part 2

When a trinomial has a leading coefficient that is not 1, and it cannot be (fully) common factored, then our factoring approach changes. This is called **factoring by decomposition**, and our process changes a little bit.

In $ax^2+bx+c...$

- Multiplies to ac .
- Adds to b .
- Numbers are ...
- Split the b term into the numbers, found above.
- Factor by grouping.

Example 1:

$$\begin{aligned} & 2x^2+5x+3 \\ = & 2x^2+2x+3x+3 \\ = & 2x(x+1)+3(x+1) \\ = & (x+1)(2x+3) \end{aligned}$$

$$\begin{aligned} \text{M: } & 2 \cdot 3 = 6 \\ \text{A: } & 5 \\ \text{N: } & 2, 3 \end{aligned}$$

$$\begin{aligned} & 4x^2+5x-6 \\ = & 4x^2+8x-3x-6 \\ = & 4x(x+2)-3(x+2) \\ = & (x+2)(4x-3) \end{aligned}$$

$$\begin{aligned} \text{M: } & 4 \cdot (-6) = -24 \\ \text{A: } & 5 \\ \text{N: } & 8, -3 \end{aligned}$$

Factoring by grouping can be extended into higher degree polynomials, especially when there are an even number of terms.

Example 2:

$$\begin{aligned} & x^3-3x^2-4x+12 \\ = & x^2(x-3)-4(x-3) \\ = & (x-3)(x^2-4) \end{aligned}$$

$$\begin{aligned} & 18x^3+9x^2+8x+4 \\ = & 9x^2(2x+1)+4(2x+1) \\ = & (2x+1)(9x^2+4) \end{aligned}$$

$$\begin{aligned} & 2ab+4ac+6bd+12cd \\ = & 2a(b+2c)+6d(b+2c) \\ = & (b+2c)(2a+6d) \end{aligned}$$

$$\begin{aligned} & 2x^3-3x^2+4x+6 \\ = & x^2(2x-3)+2(2x+3) \end{aligned}$$

Doesn't work! This can't be factored this way.

Some polynomials have a similar form to factoring quadratic trinomials. So long as the exponents are appropriate (the higher exponent is double the lower), we can trinomial factor.

Example 3:

$$\begin{aligned} & x^4 - 10x^2 + 9 \\ \text{Let } a &= x^2 \\ \text{then } a^2 &= (x^2)^2 = x^4 \\ & \underline{a^2 - 10a + 9} \quad \begin{array}{l} M: 9 \\ A: -10 \\ N: -1, -9 \end{array} \\ &= (a-1)(a-9) \\ &= (x^2-1)(x^2-9) \\ &= (x-1)(x+1)(x-3)(x+3) \end{aligned}$$

$$\begin{aligned} & 8x^6 - 65x^3 + 8 \\ \text{Let } a &= x^3; \quad a^2 = x^6 \\ & 8a^2 - 65a + 8 \quad \begin{array}{l} M: 64 \\ A: -65 \\ N: -64, -1 \end{array} \\ &= 8a^2 - 64a - a + 8 \\ &= 8a(a-8) - 1(a-8) \\ &= (a-8)(8a-1) \\ &= (x^3-8)(8x^3-1) \end{aligned}$$

As in previous courses, you should always common factor first if possible.

Example 4:

$$\begin{aligned} & 2x^4 + 8x^3 - 2x^2 - 8x \\ &= 2x(x^3 + 4x^2 - x - 4) \\ &= 2x[x^2(x+4) - 1(x+4)] \\ &= 2x(x+4)(x^2-1) \\ &= 2x(x+4)(x-1)(x+1) \end{aligned}$$