2.8 FACTORING $ax^2 + bx + c$

Factoring trinomials in the form $ax^2 + bx + c$, where $a \neq 1$, is simplified if we factor by grouping. To do this we must break up the middle term into two parts.

$$\begin{aligned} 12x^2 + 17x + 6 &= 12x^2 + 9x + 8x + 6 \\ &= (12x^2 + 9x) + (8x + 6) \\ &= 3x(4x + 3) + 2(4x + 3) \\ &= (4x + 3)(3x + 2) \end{aligned}$$

A problem arises in trying to determine how to break up the middle term. For 17x in the above example, we might have used $10x + 7x$, or $2x + 15x$, or $-3x + 20x$, etc. The decision as to which two terms must be used will be clarified if we analyse a general expansion.

$$\begin{aligned} (px + r)(qx + s) &= pqx^2 + psx + qrx + rs \\ &= pqx^2 + (ps + qr)x + rs \\ &= ax^2 + bx + c \end{aligned}$$

If we break up the middle term bx into two terms, say mx and nx, then it is clear that

$$m + n = ps + qr = b$$

and

$$m \times n = pqrs = ac$$

EXAMPLE 1. Factor $6x^2 + 5x - 6$ over the integers.

SOLUTION:

For $6x^2 + 5x - 6$, $a = 6$, $b = 5$, and $c = -6$. To factor by grouping, we will replace 5x by mx + nx where

$$m + n = b = 5$$
$$m \times n = ac = -36$$

Trial Factors:	$(18)(-2)$	$(12)(-3)$	$(9)(-4)$	$(6)(-6)$	...
Corresponding Sums:	16	9	5	0	...

Therefore, m and n are 9 and -4.

$$\begin{aligned} 6x^2 + 5x - 6 &= 6x^2 + 9x - 4x - 6 \\ &= 3x(2x + 3) - 2(2x + 3) \\ &= (2x + 3)(3x - 2) \end{aligned}$$

EXAMPLE 2. Factor $24x^2 - 14xy - 20y^2$ over the integers.

SOLUTION:

Always check for common factors.

$$24x^2 - 14xy - 20y^2 = 2[12x^2 - 7xy - 10y^2]$$

We now factor $12x^2 - 7xy - 10y^2$ by breaking $-7xy$ into two parts. $m + n = -7$ and $mn = -120$. Therefore, m and n are 8 and -15.

$$\begin{aligned} 2[12x^2 - 7xy - 10y^2] &= 2[12x^2 + 8xy - 15xy - 10y^2] \\ &= 2[(12x^2 + 8xy) - (15xy + 10y^2)] \\ &= 2[4x(3x + 2y) - 5y(3x + 2y)] \\ &= 2[(3x + 2y)(4x - 5y)] \\ &= 2(3x + 2y)(4x - 5y) \end{aligned}$$

EXERCISE 2.8

A 1. Complete the factoring.

(a) $x^2 + 10x + 16 = (x + 8)(\blacksquare)$
(b) $m^2 - 8m + 12 = (m - 2)(\blacksquare)$
(c) $n^2 - n - 2 = (\blacksquare)(n + 1)$
(d) $x^2 + 3x - 18 = (\blacksquare)(x + 6)$
(e) $x^6 - 20x^3 + 36 = (\blacksquare)(x^3 - 2)$
(f) $x^2 - x - 20 = (x - 5)(\blacksquare)$
(g) $5 + 6s + s^2 = (5 + s)(\blacksquare)$
(h) $56 - 15t + t^2 = (7 - t)(\blacksquare)$
(i) $27 - 12x + x^2 = (\blacksquare)(3 - x)$

2. If possible, determine integer values for m and n.

(a) $m + n = -8$, $mn = 15$
(b) $m + n = 5$, $mn = 4$
(c) $m + n = 7$, $mn = -18$
(d) $m + n = -1$, $mn = -12$
(e) $m + n = -2$, $mn = -10$
(f) $m + n = 3$, $mn = -28$
(g) $m + n = -7$, $mn = -18$
(h) $m + n = 6$, $mn = -27$
(i) $m + n = 7$, $mn = 15$

B 3. Factor over the integers, where possible.

(a) $6x^2 - 7x - 20$
(b) $12x^2 + 23x + 5$
(c) $22t^2 + 13t + 1$
(d) $6r^2 + 11r + 6$
(e) $6m^2 - 7m - 3$
(f) $2y^2 - y - 28$
(g) $12q^2 + 29q + 15$
(h) $20x^2 - 3x - 9$
(i) $12x^2 - 40x - 7$
(j) $3m^2 - 19m - 20$
(k) $10t^2 - 31t + 15$
(l) $10x^2 - 16x + 3$
(m) $6q^2 - 23q + 7$
(n) $4m^2 + 23m - 35$
(o) $3y^2 + 22y - 16$
(p) $3x^2 + 25x + 42$
(q) $4s^2 + 31s - 45$
(r) $20x^2 + 11x - 4$
(s) $20x^2 - 64x + 35$
(t) $36m^2 - 7m - 15$
(u) $16t^2 - 18t - 9$
(v) $15x^2 + 27x + 8$
(w) $4s^2 + 21s + 27$
(x) $8x^2 - 2x - 3$

4. Factor over the integers, where possible.

(a) $24x^2 - 2x - 2$
(b) $10x^2 + 29x - 21$
(c) $6x^2 - 27x - 15$
(d) $6x^2 + 17xy + 5y^2$
(e) $12x^2 + 13xy - 35y^2$
(f) $60m^2 + 370mn + 60n^2$
(g) $24x^2 - 47xy + 20y^2$
(h) $15t^2 + 22st + 8s^2$
(i) $2 + 2x - 84x^2$
(j) $5yx^2 + 18yx - 8y$
(k) $4x^4 + 35x^2 + 49$
(l) $2x^2 + 2xy + 2y^2$
(m) $10 + 17x + 17x^2$
(n) $15 - 44x - 20x^2$

5. Factor over the integers.

(a) $8x^2 + 38x + 45$
(b) $20y^2 + 44y - 15$
(c) $40t^2 - 47t + 12$
(d) $48x^2 + 74x + 21$
(e) $42m^2 - 51m + 15$
(f) $40x^2 + 38x + 7$
(g) $8m^2 + 46m + 63$
(h) $48y^2 - 26y + 3$
(i) $20s^2 - 29s - 33$
(j) $15 - 53x + 42x^2$
(k) $42 + t - 56t^2$
(l) $28m^2 + 107m + 99$

MICRO MATH

The following program checks your factoring by multiplication. Use the program to check your answers.

NEW

```
10 PRINT "QUADRATIC FACTOR CHECKER"
20 PRINT "(PX + M)(QX + N)"
30 PRINT "ENTER P,Q,M,N IN ORDER."
40 INPUT "P = ";P
50 INPUT "Q = ";Q
60 INPUT "M = ";M
70 INPUT "N = ";N
80 PRINT "THE POLYNOMIAL IS"
90 PRINT (P*Q);"X↑2 +("P*N+Q*M;")X
   +(";M*N")"
95 END
```

RUN

Statement 90 in the program above should be entered on one line.

MIND BENDER

A clock is stopped 1 min every 5 min. How long will it take the minute hand to complete one revolution?

360