What is the remainder when

$x^3 - 4x^2 + 7x + 8$ is divided by $\underbrace{x-3}_{3}$?

$x-3 \overline{) x^3 - 4x^2 + 7x + 8}$ R.

The Factor Theorem & Rational Zeroes Test

Recall: A number is a **factor** of another number if it divides evenly into it.

For example, 7 is a factor of 42 (because 7 x 6 = 42), but 4 is not.

Example 1: Is $(x - 2)$ a factor of $x^3 - 3x^2 + x + 2$?

$P(2) = 2^3 - 3(2)^2 + 2 + 2$

$= 8 - 12 + 2 + 2$

$= 0$

$\therefore$ $(x-2)$ is a factor.

The Factor Theorem

"if and only if"

A polynomial function $P(x)$ has a factor of $(x - k)$ iff $P(k) = 0$.

Similarly, $(jx - k)$ is a factor iff $P(^k/_j) = 0$.

$2x-3$ $P(\frac{3}{2})$

Example 2: Determine a factor of $x^3 - 7x - 6$.

$x=3 \to (x-3)$

$x=-2 \to (x+2)$

$x=-1 \to (x+1)$

$= (x-3)(x+2)(x+1)$

When factoring a quadratic...

$x^2 - 7x + 6$

$= (x-1)(x-6)$

M 6

A -7

N: -1, -6

<u>Rational Zero Test</u>

Every *rational* zero of $P(x)$ is of the form $^{p}/_{q}$, where p is a factor of the constant term and q is a factor of the leading coefficient.

<u>Example 3:</u> Determine all the *potential* roots of $f(x) = 3x^3 + x^2 - 22x - 24$.

q p

p: $\{\pm 1, 2, 3, 4, 6, 8, 12, 24\}$

q: $\{\pm 1, 3\}$

$\frac{p}{q}$: $\{\pm 1, 2, 3, 4, 6, 8, 12, 24, \frac{1}{3}, \frac{2}{3}, \frac{4}{3}, \frac{8}{3}\}$

Example 4: Factor $f(x) = 3x^3 + x^2 - 22x - 24$.

$f(3) = 0 \rightarrow (x-3)$ is a factor.

$$
\begin{array}{r|l}
 & 3x^2 + 10x + 8 \\
\hline
x-3 & 3x^3 + x^2 - 22x - 24 \\
 & -(3x^3 - 9x^2) \\
\hline
 & 10x^2 - 22x \\
 & -(10x^2 - 30x) \\
\hline
 & 8x - 24 \\
 & 8x - 24 \\
\hline
 & 0
\end{array}
$$

$3x^3 + x^2 - 22x - 24$

$= (x-3)(3x^2 + 10x + 8)$

$= (x-3)(x+2)(3x+4)$

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