

## Applications of Polynomials – Part 2

Example 1: Given the prism we looked at yesterday...

- a. Determine the value of  $x$  that yields a volume of 1430.

$$V = -4x^3 - 6x^2 + 320x + 480$$

$$1430 = -4x^3 - 6x^2 + 320x + 480$$

$$4x^3 + 6x^2 - 320x + 950 = 0$$

$$2(2x^3 + 3x^2 - 160x + 475) = 0$$

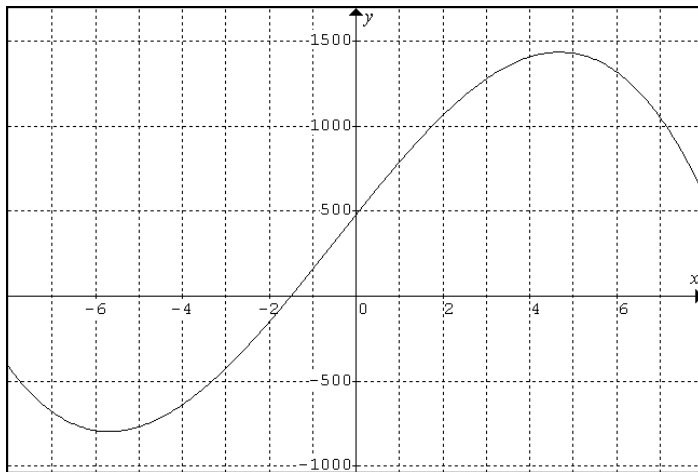
$$2(x-5)(2x^2 + 13x - 95) = 0$$

$$x = 5, 4.36, -10.86$$

The possible values for  $x$  are 5 and 4.36.

- b. Determine the maximum volume this prism can have.

Using technology, we get a maximum volume of about 1436 at about 4.7.



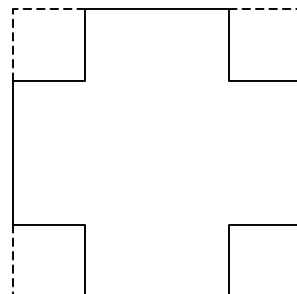
Example 2: To create a box, corners are cut from a 15 by 15 sheet of cardboard, then folded up and taped. What is the maximum volume this box can have?

Let  $x$  be the width cut from the corner.

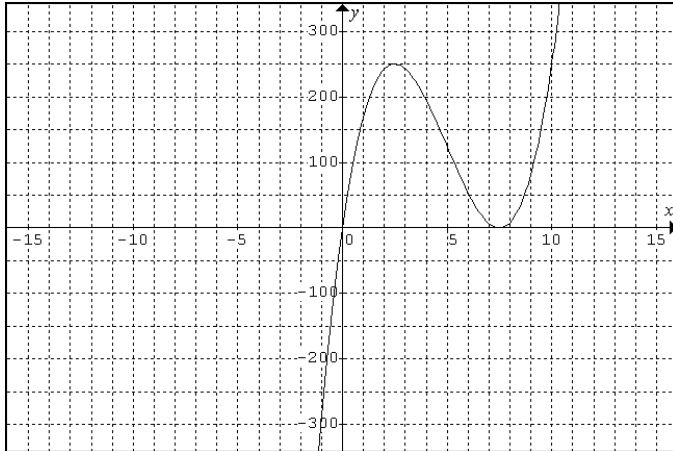
The volume of the box is:

$$V = (x)(15 - 2x)(15 - 2x)$$

$$V = 225x - 60x^2 + 4x^3$$



Using technology:



From the graph, it appears that the volume can grow without bound. However, if  $x > 7.5$ , the length & width of the box would be negative, so we focus between 0 and 7.5.

The maximum value occurs at (2.5, 250), so the largest possible volume is 250.

Example 3: A tunnel for a conveyor built has an elliptical shape. The formula for this ellipse is given by  $x^2 + 9y^2 = 9$ . Determine the largest rectangular package that can fit through this tunnel.

$2x$  is the width of the tunnel.

$2y$  is the height of the tunnel, but we need this in terms of  $x$ .

$$x^2 + 9y^2 = 9$$

$$9y^2 = 9 - x^2$$

$$3y = \sqrt{9 - x^2}$$

$$y = \frac{\sqrt{9 - x^2}}{3}$$

We want the biggest area (of the end of the package), so:

$$A = lw$$

$$A = 2x(2y)$$

$$A = 2x \left( \frac{2\sqrt{9 - x^2}}{3} \right)$$

$$A = \frac{4x\sqrt{9 - x^2}}{3}$$

To find the maximum, we once again turn to technology!

When  $x = 2.12$ , we get a maximum area of 6.

To find  $y$ , we sub 2.12 into our expression:

$$y = \frac{\sqrt{9 - 2.12^2}}{3}$$

$$y = 0.71$$

Therefore the dimensions of the package are about 4.24 ( $2x$ ) by 1.42 ( $2y$ ).

