# Special Triangles

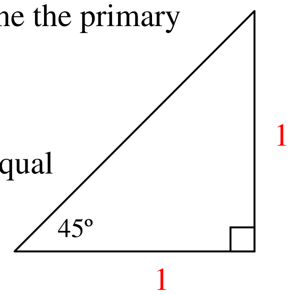
In mathematics, we try to keep things as exact as possible. For example, when you solve the equation $3x - 1 = 0$, mathematicians prefer the solution $x = \frac{1}{3}$ (which is *exact*) to the approximate solution of $x = 0.33$. Similarly, when solving $x^2 - 5 = 0$, mathematicians prefer the solution $x = \sqrt{5}$ as opposed to $x = 2.24$.

There are a handful of angles that can be expressed with exact values, such as:

$$\sin 3° = \frac{\sqrt{30} + \sqrt{10} - \sqrt{6} - \sqrt{2} - 2\sqrt{15 - 3\sqrt{5}} - 2\sqrt{5 - \sqrt{5}}}{4}$$

We'll work with some simpler expressions, but it's nice to know exact solutions exist!

Example 1: A right angled triangle contains a 45° angle. Determine the primary trigonometric ratios for this angle.




First, we note that the third angle is also 45°, so this is an isosceles triangle. Since we don't know the side lengths, we will make the equal sides 1 to keep things simple.

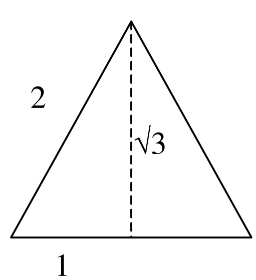
$$\tan(45°) = \frac{opp}{adj} = \frac{1}{1} = 1$$

We could use sine or cosine to find the hypotenuse, but to keep things exact, we'll use Pythagorean Theorem:

$$c^2 = a^2 + b^2$$
$$c^2 = 1^2 + 1^2$$
$$c^2 = 2$$
$$c = \sqrt{2}$$

Thus $\sin(45°) = \frac{opp}{hyp} = \frac{1}{\sqrt{2}}$ and $\cos(45°) = \frac{adj}{hyp} = \frac{1}{\sqrt{2}}$.

Similarly, you can use an equilateral triangle to show that:

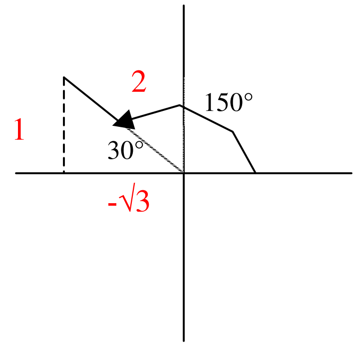



$$\sin 30° = \frac{1}{2} \quad \cos 30° = \frac{\sqrt{3}}{2} \quad \tan 30° = \frac{1}{\sqrt{3}}$$

$$\sin 60° = \frac{\sqrt{3}}{2} \quad \cos 60° = \frac{1}{2} \quad \tan 60° = \sqrt{3}$$

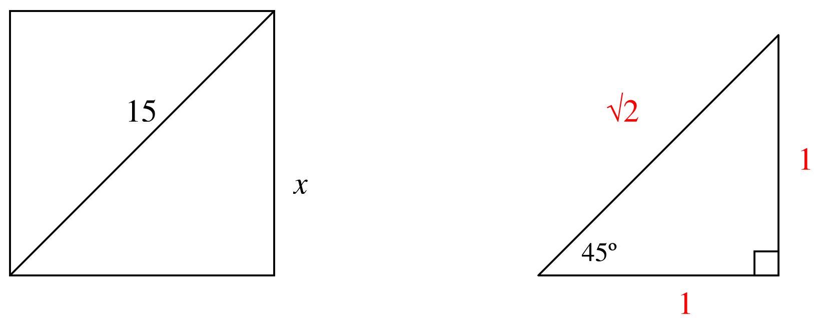
Example 2: Determine the exact value of sin150°.

First, we sketch the location of the angle, and draw a triangle in that quadrant. Noting that the supplementary angle is 30°, we label the triangle:




Thus sin150° = ½.

Example 3: The diagonal of a square-framed photograph is about 15 inches. What are the dimensions of the frame?




For this, we use similar triangles.

$$\frac{15}{\sqrt{2}} = \frac{x}{1}$$

$$x = \frac{15}{\sqrt{2}}$$

Thus the dimensions are $\frac{15}{\sqrt{2}}$ by $\frac{15}{\sqrt{2}}$ (about 10.6 by 10.6)

Practice: pg. 2 # 5, 6, 8, 9, 10, 14