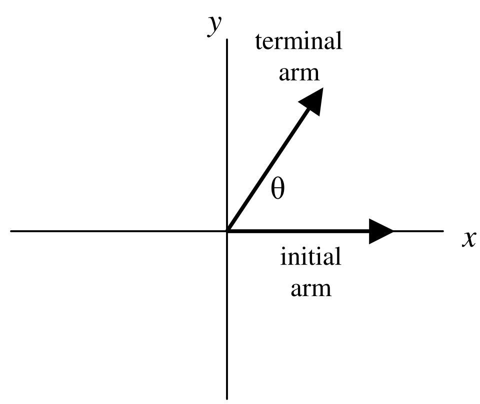
Trig Ratios and Obtuse Angles

Trigonometry has many applications outside of "solving triangles". One application is to apply trig to co-ordinate systems for the purposes of navigation.

Definition:
An angle is in **standard position** when its initial arm is located on positive x-axis.

The angle opens in the counter-clockwise direction.



Summary: If an angle is in standard position, and the terminal passes through the point (x, y), then the primary trigonometric ratios are:

$\sin\theta = \frac{y}{r}$, $\cos\theta = \frac{x}{r}$, and $\tan\theta = \frac{y}{x}$, where $r = \sqrt{x^2 + y^2}$.

When using a co-ordinate system, the location of the angle influences the sign of the ratio. For example, in the second quadrant (obtuse angles), the sine of an angle is positive, but the cosine and tangent are negative.

Example 1: The terminal arm of an angle in standard position passes through the point (-4, 3). Determine the value of the primary trigonometric ratios for this angle.

First, we determine the value of r, then substitute.

$$r = \sqrt{x^2 + y^2}$$
$$r = \sqrt{(-4)^2 + (3)^2}$$
$$r = \sqrt{16 + 9}$$
$$r = \sqrt{25}$$
$$r = 5$$

$$\sin\theta = \frac{y}{r} \qquad \cos\theta = \frac{x}{r} \qquad \tan\theta = \frac{y}{x}$$
$$\sin\theta = \frac{3}{5} \qquad \cos\theta = \frac{-4}{5} \qquad \tan\theta = \frac{3}{-4}$$

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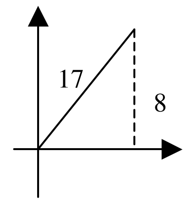
Example 2: An acute angle in standard position has a sine of $\frac{8}{17}$. Determine the values of the cosine and tangent.



Since $\sin\theta = \dfrac{y}{r}$, we know $r = 17$ and $y = 8$. To determine x, we'll use the Pythagorean theorem.

$$x^2 + y^2 = r^2$$
$$x^2 + 8^2 = 17^2$$
$$x^2 + 64 = 289$$
$$x^2 = 289 - 64$$
$$x^2 = 225$$
$$x = 15$$

$$\cos\theta = \frac{x}{r}$$
$$\cos\theta = \frac{15}{17}$$

$$\tan\theta = \frac{y}{x}$$
$$\tan\theta = \frac{8}{15}$$

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Example 3: Referring to example 1, determine the measure of the angle.

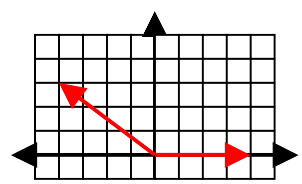
We have all three ratios available to solve for the angle, so let's see what we get:

$$\sin\theta = \frac{3}{5}$$
$$\theta = \sin^{-1}(0.6)$$
$$\theta = 37°$$

$$\cos\theta = \frac{-4}{5}$$
$$\theta = \cos^{-1}(-0.8)$$
$$\theta = 143°$$

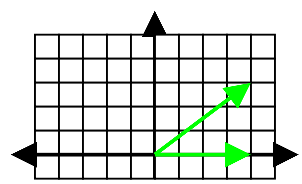
$$\tan\theta = \frac{3}{-4}$$
$$\theta = \tan^{-1}(-0.75)$$
$$\theta = -37°$$

We got three different answers! What's going on here?



If we sketch the angle, it is clearly obtuse, so the result from cosine (143°) is the correct answer.

The problem arises because there are other angles with the same ratios as the ones above. For instance, the angle passing through (4, 3) also has $\sin\theta = \frac{3}{5}$. When faced with this choice, the calculator always chooses the smaller angle – in this case, 37°.



The moral of the story is – don't trust the calculator! Think about where your angle is located.

Practice: pg. 6 #6, 12, 13