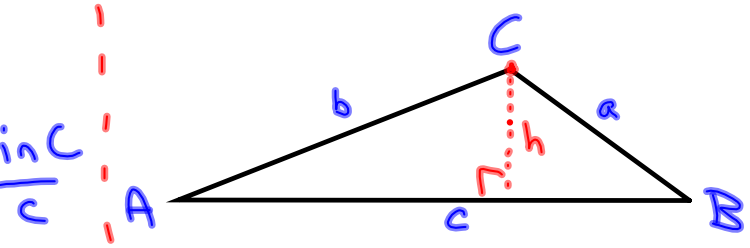
The Sine Law

Definition:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$



Proof:

$\sin A = \frac{opp}{hyp}$

$\sin A = \frac{h}{b}$

$b \sin A = h$

$\sin B = \frac{opp}{hyp}$

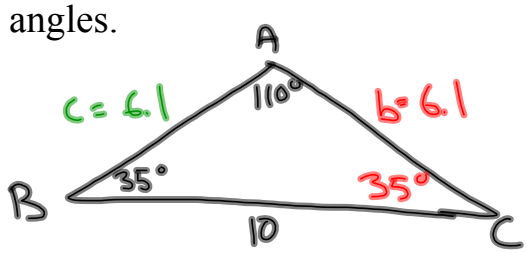
$\sin B = \frac{h}{a}$

$a \sin B = h$

$b \sin A = a \sin B$

$\frac{\sin A}{a} = \frac{\sin B}{b}$

Example 1: In triangle ABC, $a = 10$, $A = 110^o$, and $B = 35^o$. Determine the remaining sides and angles.



$C = 180 - 145 = 35°$

$\frac{\sin 35}{b} = \frac{\sin 110}{10}$

$\frac{10 \sin 35}{b} = \sin 110$

$10 \sin 35 = b \sin 110$

$\frac{10 \sin 35}{\sin 110} = b$

$6.1 \doteq b$

Since the triangle is isosceles, $c = 6.1$

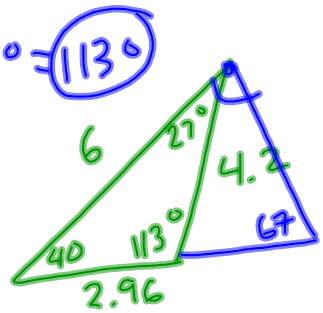
(pg. 17 #1, 2)

Example 2: In triangle ABC, $a = 4.2$, $A = 40^o$, and $b = 6$. Determine the remaining sides and angles.

$a = 4.2$ $A = 40°$

$b = 6$ $B = 67°$ → $180° - 67° = 113°$

$c = 6.25$ $C = 73°$

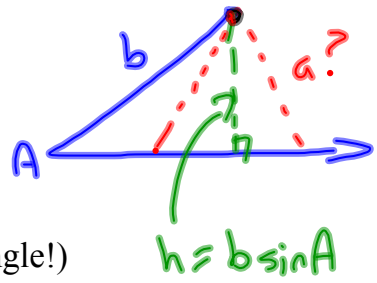


$\sin 113° =$

$\sin 67° =$

The Ambiguous Case of the Sine Law

Given sides a and b and angle A, and $a < b$, then



- If $a <$ *b*sinA, no triangle is possible.
- If $a =$ *b*sinA, one triangle is possible. (A right triangle!)
- If $a >$ *b*sinA, two triangles are possible.

$a = 4.2$ $b = 6$ $A = 40°$

→ $b \sin A$

$= 6 \sin 40°$

$= 3.85$

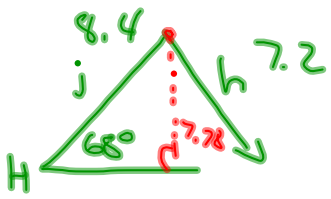
(pg 17 #5, 6, 7)

5) $\triangle HJK$, $h = 7.2$, $j = 8.4$, $\angle H = 68°$

a) $j \sin H$

$= 8.4 \sin 68°$

$\doteq 7.78$



b) Because $h < j \sin H$, no $\triangle$ is possible

$(7.2) < (7.79)$

6) $\triangle ABC$

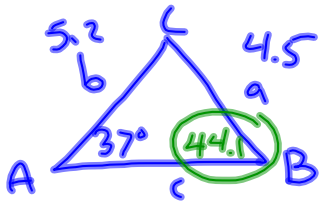
a) $b \sin A$

$= 5.2 \sin 37°$

$\doteq 3.13$

b) Since $a > b \sin A$

$(4.5) \quad (3.13)$



c) $\frac{\sin A}{a} = \frac{\sin B}{b}$

$\frac{\sin 37}{4.5} = \frac{\sin B}{5.2}$

$\frac{5.2 \sin 37}{4.5} = \sin B$

$0.695433\ldots = \sin B$

$44.1° = B$

or $B = 180 - 44.1$

$= 135.9$

Option 1: $B = 44.1^\circ$

$$\frac{\sin C}{c} = \frac{\sin A}{a}$$

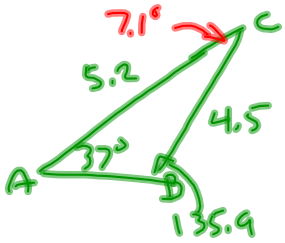
$$\frac{\sin 98.9}{c} = \frac{\sin 37}{4.5}$$

$$4.5 \sin 98.9 = c \sin 37$$

$$\frac{4.5 \sin 98.9}{\sin 37} = c$$

$$7.39 \doteq c$$

Option 2: $B = 135.9^\circ$



$$\frac{\sin 7.1^\circ}{c} = \frac{\sin 37^\circ}{4.5}$$

...

$$\frac{4.5 \sin 7.1}{\sin 37} = c$$

$$0.9242 \doteq c$$