

Problem Solving - Sine Law

Example 1: Two tracking stations, which are 100km apart, are monitoring a weather balloon. The angle of elevation from the first station to the balloon is 15° . The angle of elevation from the second station to the balloon is 10° . How high above the ground is the balloon?

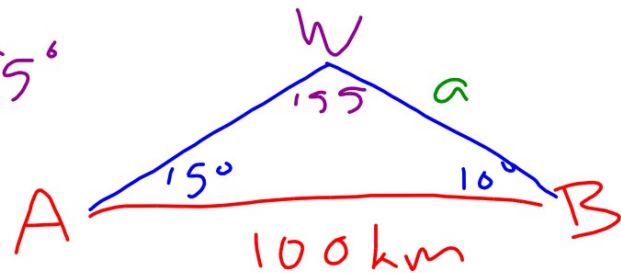
$$\angle W = 180 - 15 - 10 = 155^\circ$$

$$\frac{a}{\sin 15} = \frac{100}{\sin 155}$$

$$\frac{a}{0.2598} = \frac{100}{0.4226}$$

$$a = \frac{100}{0.4226} \times 0.2598$$

$$a = 61.2$$



$$\sin 10 = \frac{h}{61.2}$$

$$0.1736 = \frac{h}{61.2}$$

$$10.6 = h$$



Example 2: Albert and Belle are preparing to launch a weather balloon. Albert's rope is 7.8m long and makes an angle of 36° with the ground. Belle's rope is 5.9m long.

How far apart are Albert and Belle? Round to one decimal place.

$$\frac{\sin B}{b} = \frac{\sin A}{a}$$

$$\frac{\sin B}{7.8} = \frac{\sin 36}{5.9}$$

$$\sin B = \frac{7.8 \sin 36}{5.9}$$

$$B = \sin^{-1}(0.777072028\dots)$$

$$B = 51^\circ$$

$$C = 180^\circ - 51 - 36$$

$$= 93^\circ$$

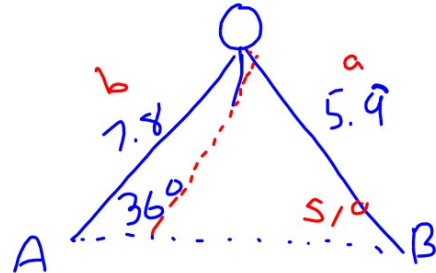
$$\frac{c}{\sin C} = \frac{a}{\sin A}$$

$$\frac{c}{\sin 93} = \frac{5.9}{\sin 36}$$

$$c = \frac{5.9 \sin 93}{\sin 36}$$

$$c = 10.02\dots$$

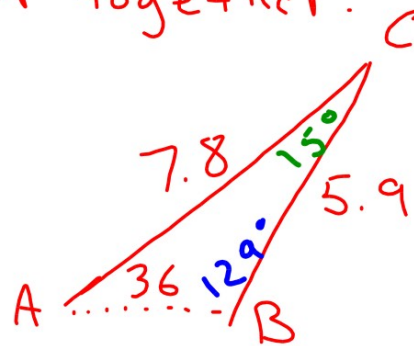
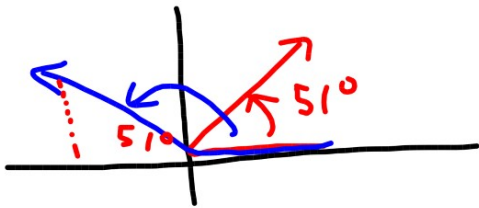
$$c \doteq 10$$



o o About 10 m
apart

What if they are closer together?

$$B = \sin^{-1}(0.7771)$$



$$\text{or } B = 180 - 51 \\ = 129^\circ$$

$$C = 180 - 129 - 36 \\ = 15^\circ$$

$$\frac{c}{\sin C} = \frac{a}{\sin A}$$

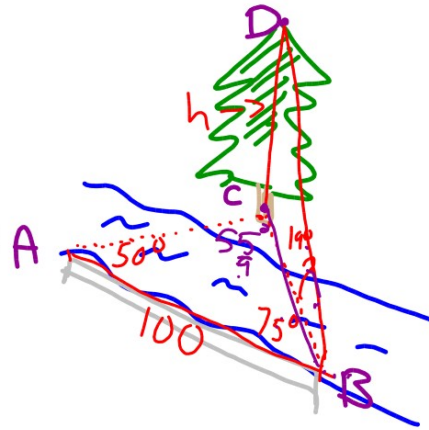
$$\frac{c}{\sin 15} = \frac{5.9}{\sin 36}$$

$$c = \frac{5.9 \sin 15}{\sin 36}$$

$$c = 2.6 \text{ m}$$

Example 3: To calculate the height of a tree across a river, the following measurements are recorded:

- Length of guardrail: 100m
- Line of sight to the base of the tree from the left end of the guardrail is at an angle of 50° ; the right end, 75° .
- Angle of elevation from the right-end of the guardrail to the top of the tree is 19° .



Determine the height of the tree.

Want side "a" in the triangle on the ground.

Need $\angle C$ to do sine law.

$$\begin{aligned} C &= 180 - 50 - 75 \\ &= 55^\circ \end{aligned}$$

$$\frac{a}{\sin 50} = \frac{100}{\sin 55}$$

$$a = \frac{100 \sin 50}{\sin 55}$$

$$a = 93.5$$

$$\begin{aligned} \tan \theta &= \frac{\text{opp}}{\text{adj}} \\ \tan 19 &= \frac{h}{93.5} \\ 93.5 \tan 19 &= h \\ \boxed{32.2} &= h \end{aligned}$$



∴ the tree is about 32.2 m tall.

Practice: pg. 18 #10 - 15