Optimization

Example 1: You have 32 ft. of decorative fencing to place around a rectangular garden. What dimensions will give you the maximum area?

$A = \ell \times w$

$A = (16 - w)w$

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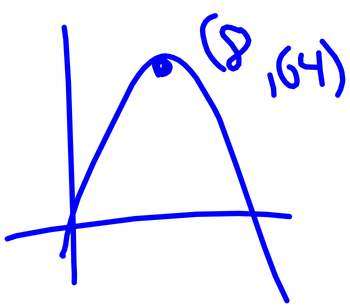


$P = 2\ell + 2w$

$32 = 2\ell + 2w$

$-2w \qquad -2w$

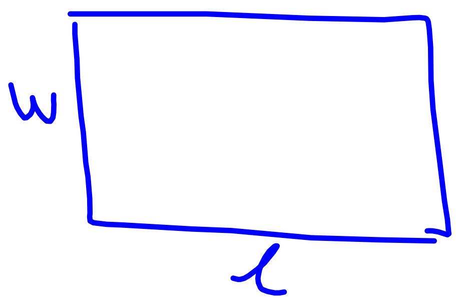
$\frac{32}{2} - \frac{2w}{2} = \frac{2\ell}{2}$

$16 - w = \ell$

$w = 8$

$\ell = 16 - 8 = 8$

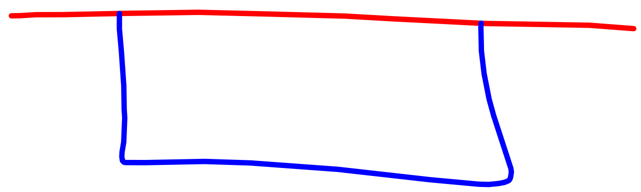
$A = 64$



$P = 32 ft$

∴ The maximum area of $64 ft^2$ occurs with dimensions of 8ft × 8ft.

<u>Example 2</u>: You decide you build the garden against the side of a building. What dimensions will give the maximum area?



$A = \ell \times w$

$= (32-2w)w$

$\downarrow$

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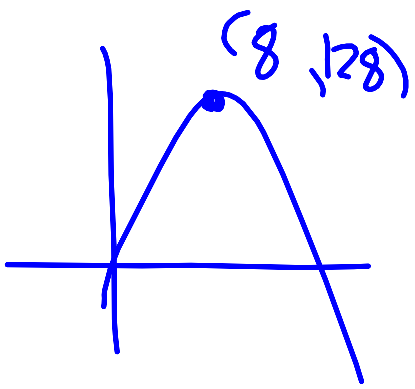
$\downarrow$



$P = \ell + 2w$

$32 = \ell + 2w$

$-2w \qquad -2w$

$32 - 2w = \ell$

$w = 8$

$\ell = 32 - 2(8) = 16$

$A = 128$

The max area of 128ft^2 occurs with dimensions of 16ft x 8ft.

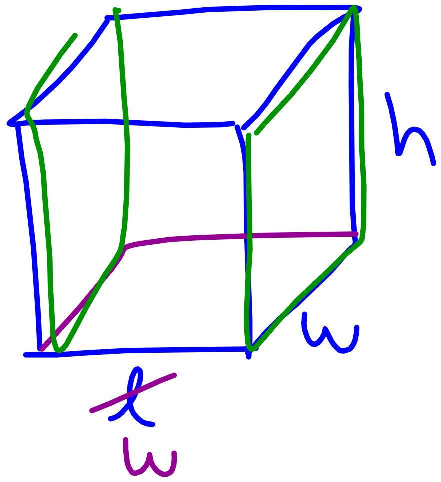
Example 3: A box will have a volume of 125ft^3. Determine the dimensions that minimize the surface area.

$V = A_{base} \times h$

$V = l \times w \times h$

Assume the base is a square.

$V = w \times w \times h$



$V = 125\,ft^3$

$SA = 2wh + 2lw + 2lh$

$= 2wh + 2w \cdot w + 2wh$

$= 2w^2 + 4wh$

$V = w \times w \times h$

$\frac{125}{w^2} = \frac{w^2 h}{w^2}$

$\frac{125}{w^2} = h$

$A = 2w^2 + 4w\left(\frac{125}{w^2}\right)$

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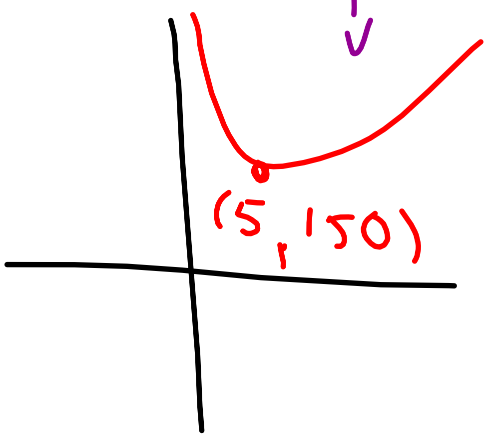


$w = 5$

$h = \frac{125}{5^2} = 5$

$SA = 150$

$\therefore$ The minimum surface area is 150ft^2 when the dimensions are $5 \times 5 \times 5$.

$\to = 355 \text{ cm}^3$

Example 4: A pop-can has a volume of 355mL. Determine the dimensions of can that minimize the amount of aluminum used.

$$V = A_{base} \times h$$

$$V = \pi r^2 h$$

$$\frac{355}{\pi r^2} = \frac{\pi r^2 h}{\pi r^2}$$

$$\frac{355}{\pi r^2} = h$$

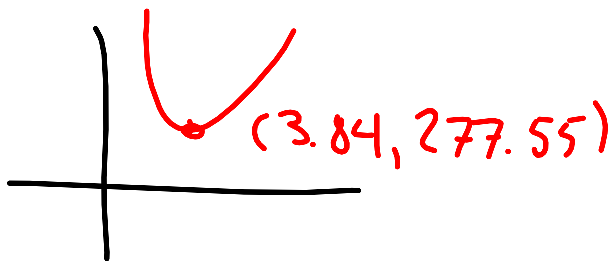
$$SA = 2\pi r^2 + 2\pi r h$$

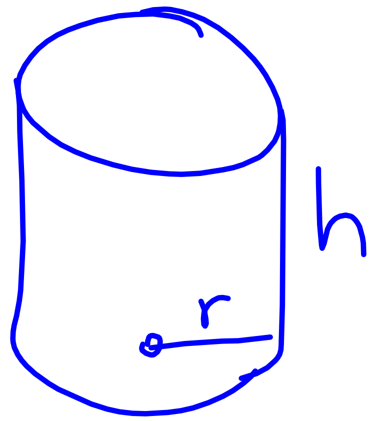
$$SA = 2\pi r^2 + 2\pi r\left(\frac{355}{\pi r^2}\right)$$

$\downarrow$

desmos

$\downarrow$





$r = 3.84$

$$h = \frac{355}{\pi(3.84)^2}$$

$\doteq 7.66$

$d = 2 \times 3.84$

$\doteq 7.68$

The minimum surface area of 277.55 cm³ occurs when the height and diameter are $\doteq$ 7.66.

Practice: → website
→ "practice 2"
→ #3, 6, 9, 10