Key Concepts

- When enclosing four sides, the maximum area for a given rectangular area is obtained by forming a square.
- When enclosing three sides, the maximum area for a given rectangular area is obtained by forming a rectangle whose length is twice its width.
- The minimum perimeter for a given rectangular area is obtained by forming a square.

Discuss the Concepts

D1. **a)** Do all rectangles with a perimeter of 10 m have the same area? Explain.

b) Does your conclusion from part a) apply to rectangles with a different perimeter? If yes, explain why. If no, provide a **counter-example**.

counter-example
- an example that contradicts a proposed truth

D2. Suppose you receive this email from a friend:

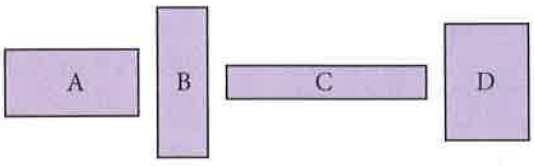
> Subject: New Garden
> To: Recipient <recipient@domain.com>
>
> Hi;
> I'm helping my mom make a new garden. She wants it as big as possible. I've got 10 m of chicken wire for fencing. Can you help me? Do you think I should back it against the garage? Any suggestions?
> Thanks!

Discuss an appropriate response to your friend.

Practise A

Use this information to answer questions 1 and 2.

These rectangles all have the same perimeter.

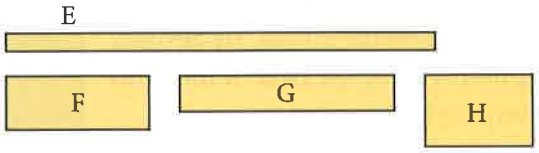


1. Order the rectangles from greatest to least area.

2. Is it possible to draw a rectangle with the same perimeter but with a greater area than these rectangles? Explain using words and a diagram.

Use this information to answer questions 3 and 4.

These rectangles all have the same area.



3. Order the rectangles from least to greatest perimeter.

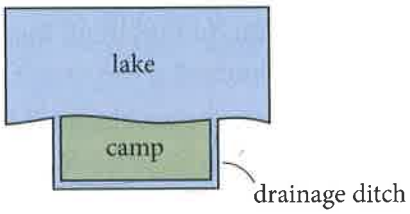
4. Is it possible to draw a rectangle with the same area that has a smaller perimeter than any of the given rectangles? Explain using words and a diagram.

Apply B

5. a) Farmer Fran has 30 segments of 2-m fence rails to build a pen for her sheep. What are the dimensions of the pen with the maximum possible grazing area for the sheep, assuming the fence rails cannot be cut?

b) How does your answer change if the fence rails can be cut? How much additional area does this provide?

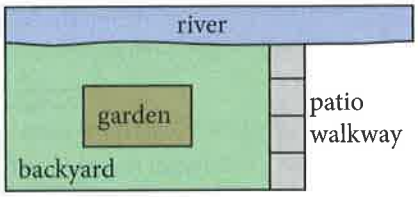
6. A scout camp is being built on the shore of a lake. The scouts are digging a narrow, 60-m long drainage ditch to surround the rectangular camp on three sides, as shown.



Determine the length and width of a ditch that will provide the maximum camp area. Use words and diagrams to show your reasoning.

Use this information to answer questions 7 to 10.

Ashraf is building a garden in his large backyard. He has 144 ft of fencing, with which to enclose his garden. He also has the option of using the river or the patio walkway (or both) instead of fencing.



7. Ashraf decides to build the garden somewhere in the middle of the backyard, as shown above. What are the dimensions of the rectangular garden with maximum area? What is the maximum area in this case?

8. Ashraf decides to build the garden against *either* the river *or* the patio walkway. What are the dimensions of the rectangular garden with maximum area? What is the maximum area in this case?

9. Ashraf decides to build the garden against *both* the river *and* the patio walkway. What are the dimensions of the rectangular garden with maximum area? What is the maximum area in this case?

10. a) Order the three gardens from questions 7 to 9 from greatest to least area.

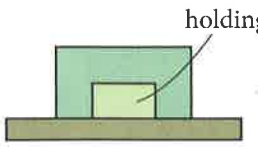
b) Which garden design do you think Ashraf should choose? Why?

11. Julia is an electronics technologist. She is designing a circuit board that has an area of 40 cm^2. It must fit snugly inside a computer console.

a) What dimensions of the circuit board will have a minimum perimeter?

b) How would your answer change if the circuit board has to fit in a slot which is only 5 cm wide? Justify your answer.

12. A marine biologist wants to use netting to build a rectangular holding pen by the beach to study marine wildlife. The pen must have an area of 120 m^2.



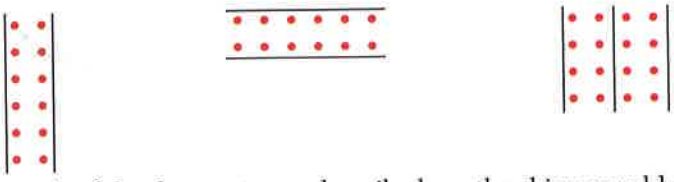
a) Determine the dimensions of the pen that require the least amount of netting.

b) Determine the total length of netting required to build the pen from part a).

Chapter Problem

13. The lift line area of Horstman Glacier is not very large and can become quite crowded. Riders wait for and ride the T-bar lift in pairs. Rail barriers are placed to organize the waiting riders.

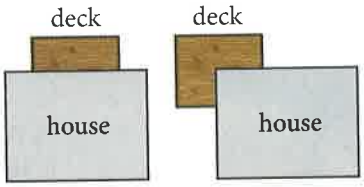
Set-Up 1	Set-up 2	Set-Up 3
One very long line.	One very wide line.	Multiple lines that arrange the riders in a square.



a) For each of the three set-ups, describe how the skiers would proceed through the waiting area. Include diagrams to help you explain.

b) Which lift line set-up do you think is best for organizing people who are waiting for the T-bar lift? Explain your reasoning.

14. Angelina wants to build a 140-ft^2 deck against her house. She is considering the options shown.



a) Assume Angelina does not have to install railing along the sides of the house. Which option minimizes the amount of railing that Angelina needs to enclose her deck?

b) What is the minimum length of railing needed?

c) What other considerations might Angelina think about when choosing her design?

Extend C

15. Ryan has found three additional support stakes at the campground. Can Ryan make a swimming area with a greater area than the rectangular one if he uses the stakes to make a different shape with the buoy rope?

a) Determine which shape maximizes the swimming area.

b) What is the maximum swimming area that can be obtained?

c) Compare your results with those of several classmates. What shapes seem to produce the greatest area?

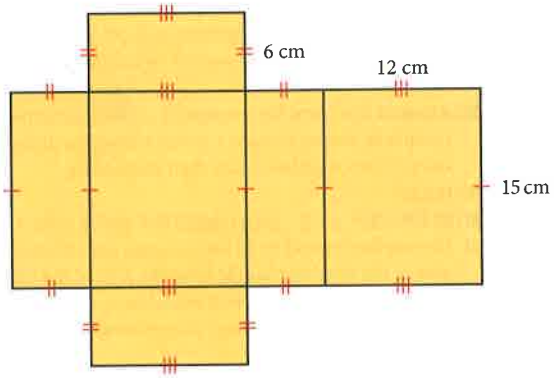
16. Elias bought 30 cm of flexible plastic edging to enclose a flowerbed. He can bend the edging into any shape. He is considering an equilateral triangle, a square, and a circle as possible shapes.

a) Predict which of these shapes will maximize the area of the flowerbed.

b) Use a graphing calculator or dynamic geometry software to test your prediction in part a).

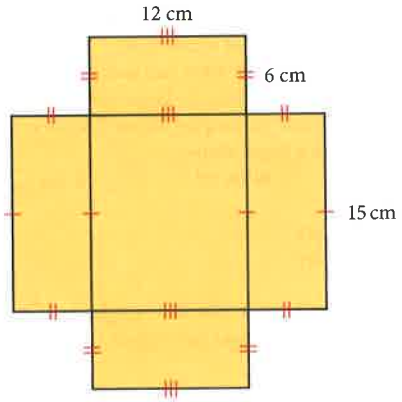
1.3 Surface Area, pages 26–35

1. a)



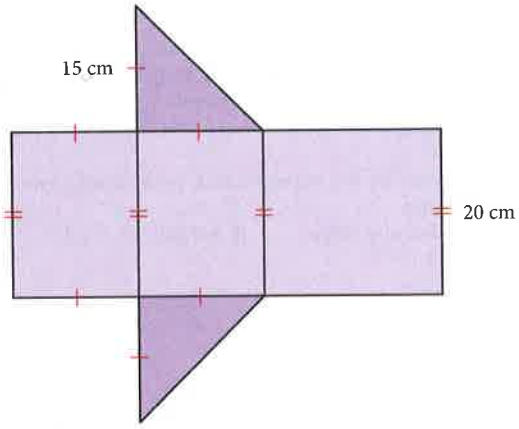
b) 684 cm^2

2. a)

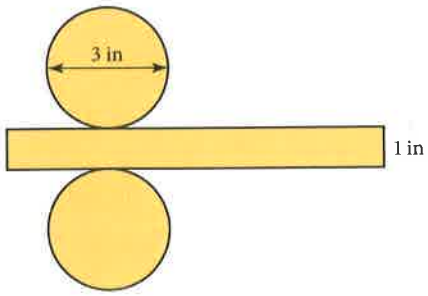


b) 504 cm^2

3. 1249 cm^2



4. 24 $in.^2$



5. 396 cm^2

6. 2376 cm^2

7. 0.42 m^2

8. 16 000 mm^2 or 160 cm^2

9. a) 76 800 mm^2 or 768 cm^2

b) One box of 16 balls requires less packaging than eight boxes of 2 balls (1280 cm^2). This makes sense because when all 16 balls are in one box there is no packaging between the balls.

10. c) 450 m^2

d) They are the same.

12. a) 13.7 m^2

b) 10.7 m^2

13. a) 1200 m^2

b) 1.9 m

14. 348.5 cm^2

15. New box is a square-based prism with sides 80 mm in length and a height of 160 mm. The golf balls are stacked in a two by two by four arrangement. The surface area of new box is 640 cm^2.

16. $374.50

1.4 Optimize Perimeter and Area, pages 36–45

1. D, A, B, C

2. Yes. A square would have the same perimeter but a greater area.

3. H, F, G, E

4. Yes. A square would have the same area but a smaller perimeter.

5. a) 14 m by 16 m

b) 15 m by 15 m; 1 m^2 is added

6. sides perpendicular to lake: 15 m, side opposite lake: 30 m

7. 36 m by 36 m; 1296 m^2

8. sides perpendicular to river: 36 m, side opposite river: 72 m; 2592 m^2

9. 72 m by 72 m; 5184 m^2

10. a) Garden against river and walkway, garden against river or walkway, garden in middle of yard.

b) Answers may vary. For example: The garden against the river and walkway, because it will give him the largest area within his fencing constraint.

11. a) 6.3 cm by 6.3 cm **b)** 5 cm by 8 cm

12. a) sides perpendicular to beach: 7.7 m, side opposite beach: 15.5 m

b) 31 m

13. Answers may vary. For example:

a) Set-Up 1: Skiers would line up and enter the T-bar lift in pairs, the first pair going first. Set-Up 2: Skiers would line up and enter the T-bar lift in pairs from the side of the line. Set-Up 3: pairs of skiers alternate entering the T-bar lift from each of the lines.

b) Set-Up 3 is best because it has the greatest area with the smallest perimeter, so more people can wait for the lifts while using the least amount of fencing.

14. a) Deck built against one side of the house.

b) 33.5 ft

c) Answers may vary. For example: Where the outside door is located, the feasibility of the layout of the deck, the view from the deck.

15. a) half of a dodecagon **b)** 13 995 ft^2

c) Answers may vary. For example: Shapes resembling semi-circles seem to produce the greatest area.

16. a) Predictions may vary.

b) circle maximizes the area