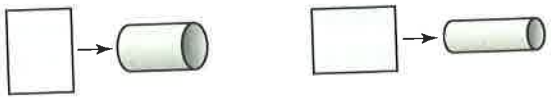
Key Concepts

- The minimum surface area for a given volume of a square-based rectangular prism occurs when the height is equal to the side length of the base.
- The minimum surface area for a given volume of a cylinder occurs when the height is equal to the diameter.
- There are a number of manipulative, technological, and algebraic tools and strategies that are useful when optimizing volume or surface area of prisms and cylinders.

Discuss the Concepts

D1. Two square-based prism shaped boxes both have a volume of 48 cm^3. Can it be concluded that these boxes have the same surface area? If yes, explain why. If no, provide a counter-example.

D2. Suppose you use two identical standard sheets of paper. You roll one lengthwise to form a cylinder and you roll the other widthwise to form a cylinder.



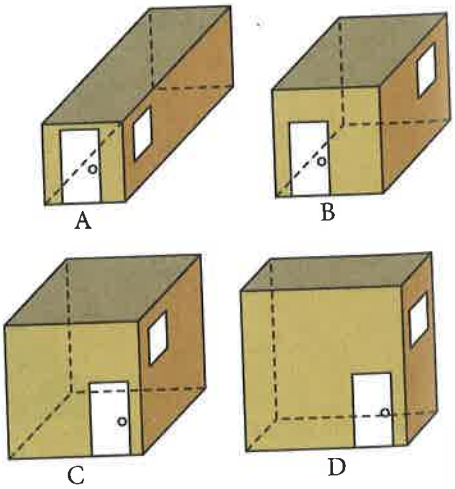
a) Suppose that the cylinders are popcorn containers (with a bottom). Which do you think would hold more popcorn? Explain why you think so.

b) Test your conjecture.

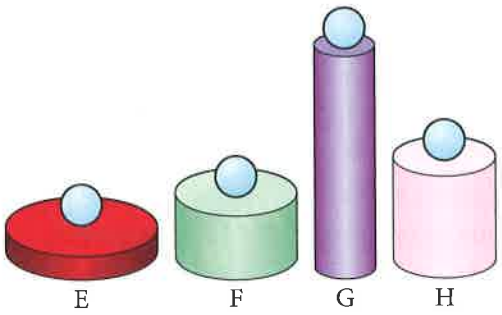
D3. Do the containers in question D2 have the same surface area? Discuss with a partner.

Practise A

1. Each shed has the same volume. Without measuring, order these sheds from minimum to maximum surface area. Do not include the bottom of the sheds. Explain your reasoning.



2. Each cylindrical container has the same surface area.



Without measuring, order these containers from maximum to minimum volume. Explain your reasoning.

Apply B

3. A candy store, in the shape of a square-based prism is to have a volume of 1000 m^3. Determine the dimensions of the store with the minimum surface area.

4. Refer to question 3. How would your answer change if the minimum surface area did not include the bottom of the building? Explain.

5. A slipper manufacturer is releasing a new product called Rocco and Biff Warm 'n Fuzzies.

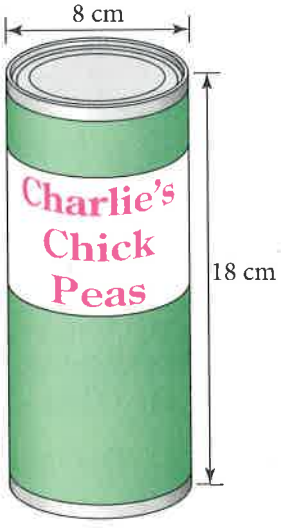
The slippers are to be shipped in boxes in the shape of square-based prisms, 50 pairs to a box. The slippers can be arranged different ways, but each slipper requires 11 000 cm^3 of space.

a) Determine the volume of each shipping box.

b) What are the dimensions of the box with a minimum surface area?

c) Sketch the box and label its dimensions.

6. Charlie's Chick Peas come in cans as shown.



a) Determine the volume of one can.

b) Could Charlie save money on packaging materials by altering the design of his can? Explain.

c) Determine the maximum amount Charlie can save on materials without reducing the volume of the container. Express your answer as a percent.

7. A soup can is to hold 600 mL of soup. Determine the dimensions of the can with minimum surface area.

8. A producer of nutrition bars is designing a bar that will just fit inside a package in the shape of an equilateral triangular-based prism.

a) Determine the dimensions of a 350-mL bar that requires a minimum amount of packaging material.

b) Describe the tools and strategies you used to solve this problem, and any assumptions you made.

Achievement Check

9. Jessie has 24 m^2 of wood with which to build a tree house.

Jessie plans to build the tree house in the shape of a square-based prism. What is the maximum volume she can enclose?

10. Refer to question 9. How much additional volume can be added to the tree house in each case?

i) Jessie leaves the roof open.

ii) Jessie leaves the roof and back wall (facing the tree trunk) open.

Explain your solution in each case, including any assumptions that you made.

11. Find an object in your classroom or at home that uses geometrically shaped packaging, such as a tissue box or a can of food.

a) Take measurements and determine the volume of the object.

b) Calculate the surface area of the object.

c) Is the object packaged optimally? Explain.

d) Determine the dimensions of a package having the same shape and volume but with a minimum surface area.

e) Suggest reasons why objects in stores are not all packaged optimally.

12. When designing an apartment or office building, the shape of the rectangular prism with minimum surface area is a cube. Many buildings in towns and small cities are close to cube-shaped. However, in the downtown areas of many large cities, this shape is less common. What type of shape is more common in cities? Why do you think this is so?

13. Can you think of a situation in which a retailer may not wish to optimize the volume of a package or container for a given surface area? Provide an example and explain why they might not wish to do so.

Extend C

14. A tent is in the shape of a triangular prism when constructed.

a) Use *The Geometer's Sketchpad*® to show why the base of the triangular face of the tent must always be double the height in order to preserve the shape of the face.

b) Use algebraic and geometric reasoning to prove this fact.

15. For a given surface area, which type of container can hold the greatest volume: a square-based prism or a cylinder? How did you determine the answer?

16. A bar of soap is in the shape of a rectangular-based prism, with dimensions 1 cm by 3 cm by 6 cm. Determine the dimensions of a cylindrical container that will be just large enough to hold 12 bars of soap. Describe how the bars should be stacked inside the container.

17. A cylindrical can of speciality candies has been designed to minimize the surface area. It is packaged in a rectangular box that just fits the can. Does the box also have the minimum surface area for the volume enclosed? Justify your answer.