

Problem Solving with Quadratics (1)

Example 1: After a particularly frustrating semester, a math student (who wishes to remain anonymous) “chucks” his or her binder off the Niagara Escarpment. (Note: I do not recommend this to anyone because (a) littering is illegal and (b) you might squish a chipmunk or something.)

The height of the binder, h , in meters, t seconds after it has been thrown is: $h = -5t^2 + 5t + 45$.

a) Use Desmos to graph this relationship.

For Desmos to understand the equation, we first change our variables.

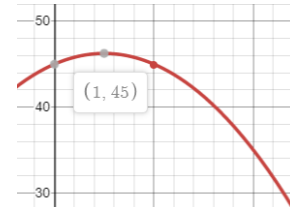
Let x be the time since the binder was released, let y be the height above the ground.

$$y = -5x^2 + 5x + 45.$$



b) How high above the ground is the binder 1 second after the binder is thrown?

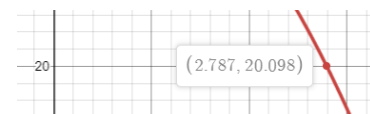
We want the time to be 1 second, so click on the curve to get a point, and drag it until you get “1” as the x -coordinate of the point. We find the point (1, 45).



So at 1 second, the binder is **45 meters** above the ground.

c) The trees below the escarpment are about 20 meters high. At what time will the binder hit the top of the trees?

This time we know the height, so we want $y = 20$. Click on the curve and follow it until you get as close to 20 as you can. When I tried it, I got the point (2.787, 20.098).



Rounding off, it takes about **2.8** seconds for the binder to hit the tops of the trees.

If you want to make this more accurate, you can add the equation $y = 20$ into Desmos, and then click on the point where it hits the parabola. Rounding, off, we still get **2.8 seconds**.



Example 2: The “Quadratics Cup” is a new coffee shop that sells various coffee-related items, but with a quadratics twist (like an equation written on your cup, or your caramel drizzle in the shape of parabolas, etc)

The profit of this new coffee shop can be described by the equation $P = -4n^2 + 64n - 112$

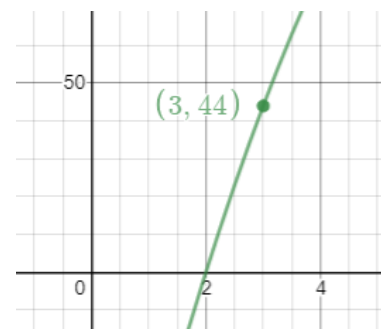
Where P represents profit in hundreds of dollars and n represents the number of customers, in thousands.

a) How much profit is made when 3000 customers are served in a day?

Since n is in *thousands*, $n = 3$ represents 3000 customers.

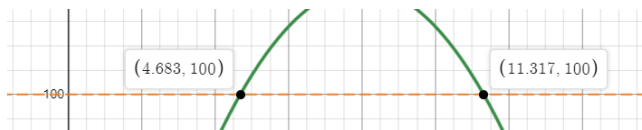
Using the graph, we get the point (3, 44).

Since profit is in *hundreds* of dollars, the coffee shop makes $44 \times \$100 = \4400 .



b) How many customers must be served per day to make a profit of at least \$10,000?

We want the profit to be at least \$10,000. Since P is in *hundreds* of dollars, we divide by 100 to get $P = 100$.



Using the graph, there are two spots where the parabola hits 100: (4.683, 100) and (11.317, 100).

Since n is in *thousands* of people, this means that if the coffee shop serves between **4683** and **11 317** people, they will make over \$10,000 in profits.