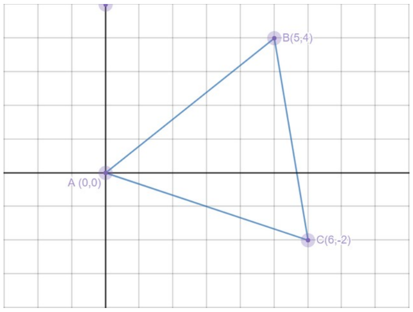
MPM 2D
Mr. Kempe

Class Notes
Unit 2, Lesson 8

Midpoints and Medians

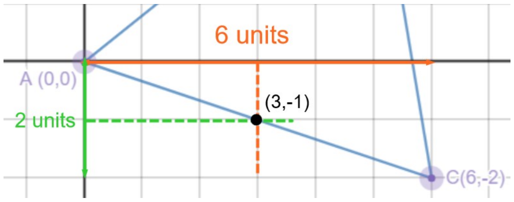
In this section of the unit we will take our understanding of lines and extend them to representing and working with geometric figures. For example, if we take a triangle and place it on a co-ordinate grid, we can label each vertex (corner) as a point on the plane.



The triangle ABC, shown right, has vertices A(0,0), B(5,4), and C(6, -2).

Example 1: Determine the **midpoint** of each side of the triangle.

The midpoint is the halfway point of each line segment. We'll start with side AC.



Horizontally, it's 6 units from A to C, so the midpoint must be 3 units from each endpoint.

Vertically, it drops 2 units from A to C, so the midpoint must be 1 unit lower than A.

Thus the midpoint is **(3, -1)**.

Because we are going halfway between the points, we are really *averaging* the x- and y-co-ordinates of each point.

Midpoint Formula

The midpoint between points A and B is given by $M_{AB}=\left(\frac{x_A+x_B}{2},\frac{y_A+y_B}{2}\right)$.

Example 2: Verify the midpoint of AC with the formula, and determine the midpoints of AB and BC.

$$M_{AC}=\left(\frac{x_A+x_C}{2},\frac{y_A+y_C}{2}\right)$$
$$M_{AC}=\left(\frac{0+6}{2},\frac{0+(-2)}{2}\right)$$
$$M_{AC}=\left(\frac{6}{2},-\frac{2}{2}\right)$$
$$M_{AC}=(3,-1)$$

$$M_{AB}=\left(\frac{x_A+x_B}{2},\frac{y_A+y_B}{2}\right)$$
$$M_{AB}=\left(\frac{0+5}{2},\frac{0+4}{2}\right)$$
$$M_{AB}=\left(\frac{5}{2},\frac{4}{2}\right)$$
$$M_{AB}=\left(\frac{5}{2},2\right)$$

$$M_{BC}=\left(\frac{x_B+x_C}{2},\frac{y_B+y_C}{2}\right)$$
$$M_{BC}=\left(\frac{5+6}{2},\frac{4+(-2)}{2}\right)$$
$$M_{BC}=\left(\frac{11}{2},\frac{2}{2}\right)$$
$$M_{BC}=\left(\frac{11}{2},1\right)$$

It works! :)

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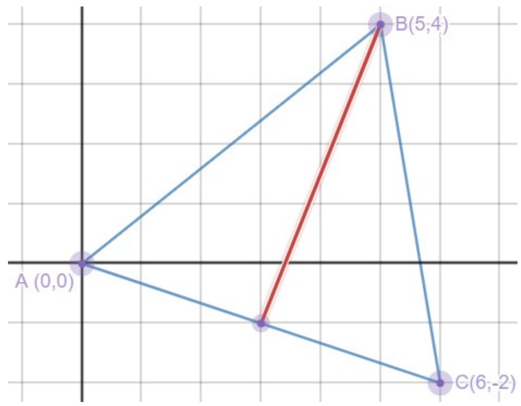


Definition: The **median** of a triangle is a line segment that runs from a vertex to the midpoint of the opposite side of the triangle.

Example 3: Determine the equation of the median line passing through point **B**.

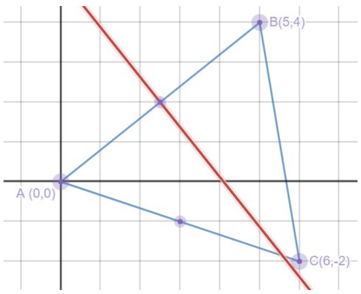
We know the co-ordinates of B (5, 4) and its midpoint M_{AC} (3, -1), so we're all set to go!

$$m=\frac{-1-4}{3-5}=\frac{-5}{-2}=\frac{5}{2}$$

$$y=mx+b$$
$$4=\left(\frac{5}{2}\right)(5)+b$$
$$4=\frac{25}{2}+b$$
$$4-\frac{25}{2}=b$$
$$-\frac{9}{2}=b$$

The equation of the median is $y=\frac{5}{2}x-\frac{9}{2}$.

Definition: A **perpendicular bisector** is a line that is perpendicular to a line segment and passes through the segment's midpoint.



Example 4: Determine the equation of the line that is the perpendicular bisector to side AB.

We know the midpoint of AB is $\left(\frac{5}{2},2\right)$. But that's the only point on this new line we know… how to we get the slope?

Recall: Two lines are perpendicular if their slopes are **negative reciprocals**.

If we find the slope of AB, we can then identify the slope of the bisector.

$m=\frac{4-0}{5-0}=\frac{4}{5}$; the negative reciprocal of this is $-\frac{5}{4}$.

With the slope $\left(-\frac{5}{4}\right)$ and a point $\left(\frac{5}{2}, 2\right)$, we are ready to create the equation of the line.

$$y = mx + b$$
$$2 = -\frac{5}{4}\left(\frac{5}{2}\right) + b$$
$$2 = -\frac{25}{8} + b$$
$$2 + \frac{25}{8} = b$$
$$\frac{41}{8} = b$$

The equation of the line is $y = -\frac{5}{4}x + \frac{41}{8}$.

Practice: pg. 79 # 2ac, 4ac, 5, 6, 9, 12, 13ab, 15*