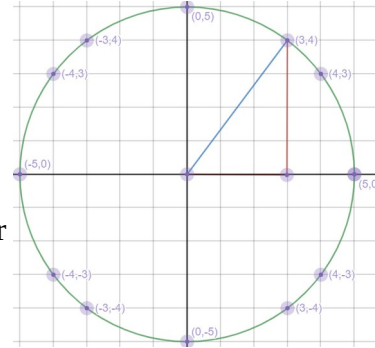


## Equations of Circles

Desmos Activity: Draw a circle of radius 5 units on a grid. Identify as many integer points as you can on the circle!

Note that for all of these points (except for the intercepts) we can draw a right-angled triangle. The hypotenuse is always 5 (because that's the radius) and the co-ordinates give us the lengths of the other side. Therefore, for all points  $(x, y)$  on the circle, the Pythagorean Theorem holds:

$$x^2 + y^2 = 5^2$$



If we replace the 5 with  $r$  (for radius), then the general equation of a circle **centered at the origin** is:  
 $x^2 + y^2 = r^2$  .

Example 1: Determine the equation of a circle that is centered at the origin and...

a. Has a radius of 10.

$$x^2 + y^2 = r^2$$

$$x^2 + y^2 = 10^2$$

$$x^2 + y^2 = 100$$

b. Passes through the point (1, 6).

$$x^2 + y^2 = r^2$$

$$1^2 + 6^2 = r^2$$

$$1 + 36 = r^2$$

$$37 = r^2$$

$$\sqrt{37} = r$$

$$x^2 + y^2 = r^2$$

$$x^2 + y^2 = (\sqrt{37})^2$$

$$x^2 + y^2 = 37$$

Example 2: The boundary of an "invisible fence" for a dog is given by the equation  $x^2 + y^2 = 225$ , where the transmitter is at the origin, and distances are measured in feet.

a) What is the radius of the boundary?

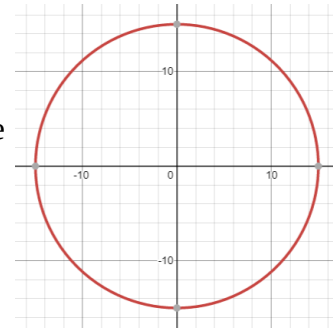
$$r^2 = 225$$

$$r = \sqrt{225}$$

$$r = 15$$

b) Draw a rough sketch of the boundary.

c) The neighbour's cat, Fluffy, is sleeping at co-ordinates  $(-8, 14)$ . Is the cat inside or outside of the boundary?



On our sketch, the point  $(-8, 14)$  is *near* the boundary. To check it exactly, we'll substitute our point into the equation:

$$\begin{aligned}x^2 + y^2 &= 225 \\(-8)^2 + (14)^2 &= 225 \\64 + 196 &= 225 \\260 &\neq 225\end{aligned}$$

Since  $260 > 225$ , the cat lies outside the boundary (and hopefully is safe from the dog).

Practice: pg. 92 #2a, 3a, 4, 5, 8, 10, 13, 19