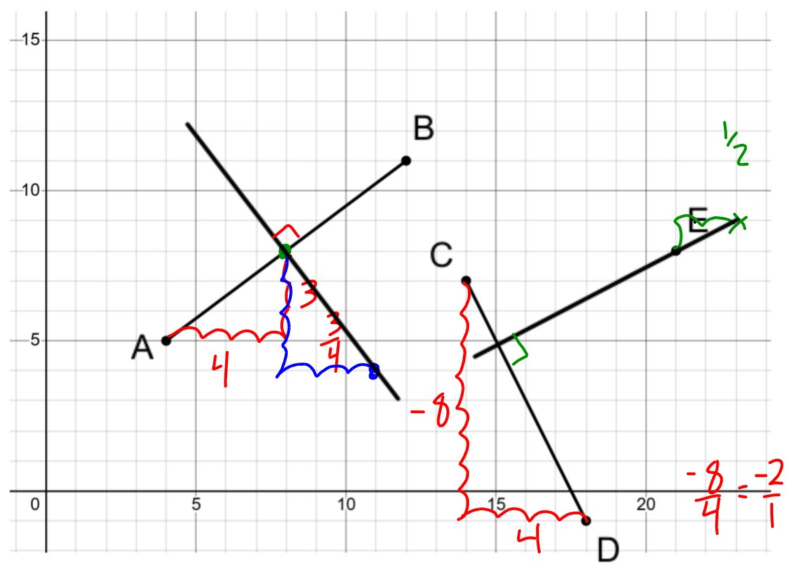
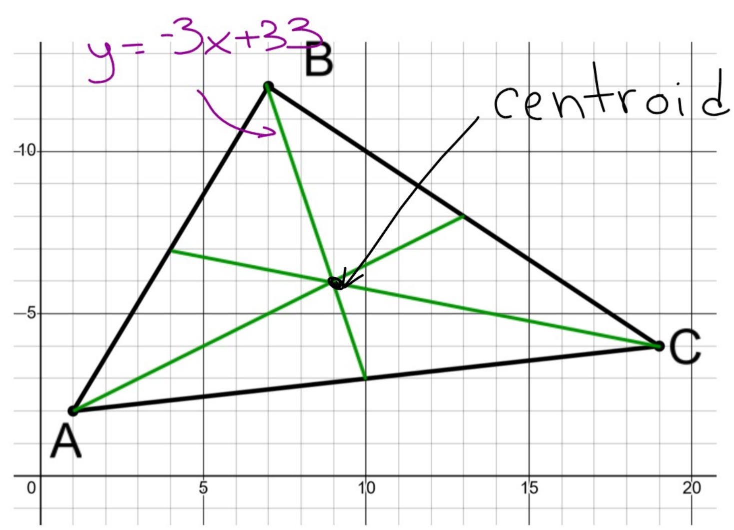
Circumcenter, Orthocenter, Centroid - Investigations

1. Add a perpendicular bisector to AB. → midpoint ✓ and perpendicular slope
2. Draw an altitude from E to CD. → perpendicular slope, drop line



Investigation 1

1. Add midpoints to each side.
2. Draw the three **medians.**
3. What do you notice?



The centroid is the balance point of the triangle.

1. <u>Centroid</u> A(1,2) B(7,12) C(19,4)

Median from B to AC

-> Need the midpoint of AC $\left(\frac{1+19}{2}, \frac{2+4}{2}\right) = (10,3)$

-> Need the slope from B to (10,3)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-12}{10-7} = \frac{-9}{3} = -3$$

Equation of the line:

$$y = mx + b$$
$$3 = -3(10) + b$$
$$3 = -30 + b$$
$$33 = b$$

$$\boxed{y = -3x + 33}$$

Repeat with another median.

e.g. A to BC is $y = \frac{1}{2}x + \frac{3}{2}$

Solve the intersection of $y = -3x+33$ and $y = \frac{1}{2}x + \frac{3}{2}$

Substitution: $2\times(-3x+33) = \left(\frac{1}{2}x + \frac{3}{2}\right)\times 2$

$$-6x + 66 = x + 3$$
$$-7x = -63$$
$$\boxed{x = 9}$$

$y = -3(9) + 33$

$\boxed{y = 6}$

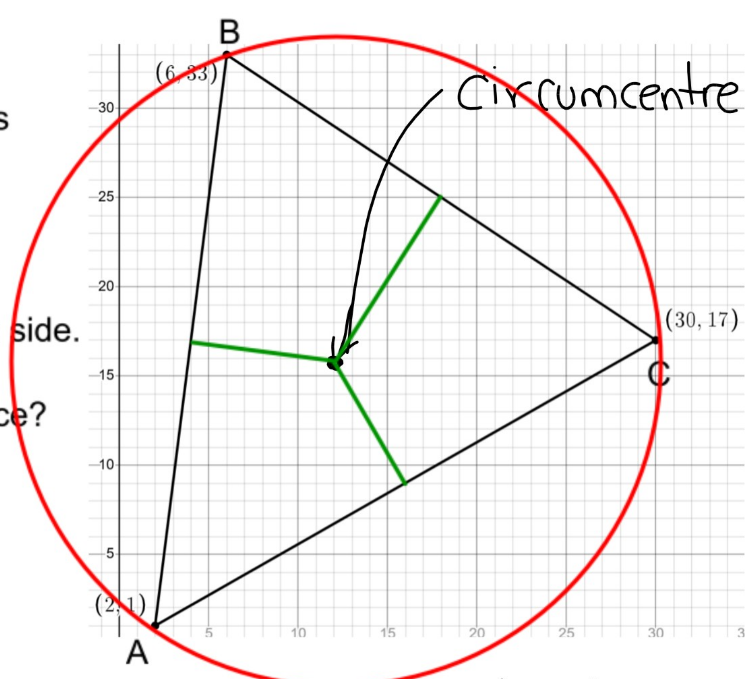
(9,6)

Investigation 2

1. Add the midpoints to each side.

2. Draw the **perpendicular bisector** on each side.

3. What do you notice?



The circumcentre is equidistant from all three vertices.

2. Circumcentre

$A(2,1)$ $B(6,33)$ $C(30,17)$

Perpendicular Bisector of AB.

Need midpoint of AB -> $(4,17)$

Need the slope of AB -> $\frac{8}{1}$

Need the perpendicular slope -> $-\frac{1}{8}$

Make the equation:

$$y = mx + b$$
$$17 = -\frac{1}{8}(4) + b$$
$$-\frac{35}{2} = b$$

$$\boxed{y = -\frac{1}{8}x - \frac{35}{2}}$$

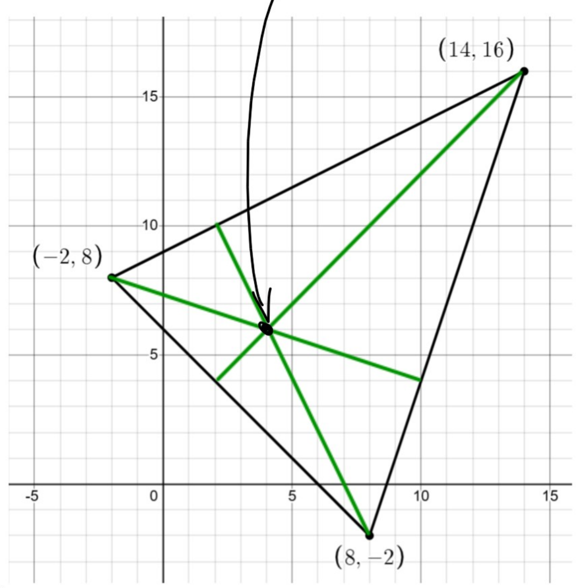
Repeat (get another line) and solve the system!

Investigation 3

1. Draw an altitude (triangle height) from each vertex to the opposite side.

2. What to you notice?

orthocentre



The orthocentre is where the three heights intersect.

3. Orthocenter $A(-2,8)$ $B(14,16)$ $C(8,-2)$

Altitude from B:

-> Need slope of AC -> $m=-1$

-> Need perpendicular slope -> 1

-> Make the equation with the slope and point B -> $y=x+2$

-> Repeat (get another line) and solve!