

## Factoring Extensions

We can use the techniques for factoring trinomials (simple trinomial, decomposition) on expressions that don't quite look like trinomials!

Example 1: Use trinomial factoring on non-quadratics.

a) Factor  $x^2 - 13x + 36$

**M:** 36 **A:** -13 **N:** -9, -4

$$= (x - 9)(x - 4)$$

b) Factor  $x^4 - 13x^2 + 36$

This trinomial has higher exponents that we are used to. To show it has the same pattern as a regular trinomial, we'll make a substitution:

Let  $a = x^2$ , which means that  $a^2 = (x^2)^2 = x^4$ .

$$x^4 - 13x^2 + 36$$

$$= a^2 - 13a + 36$$

$$= (a - 9)(a - 4)$$

$$= (x^2 - 9)(x^2 - 4)$$

$$= (x - 3)(x + 3)(x - 2)(x + 2)$$

This fits our normal pattern. **M:** 36 **A:** -13 **N:** -9, -4

Now that we've factored, we put  $x^2$  back in.

Two differences of squares.

c) Factor  $(3k - 1)^2 - 13(3k - 1) + 36$

Same idea! Let  $a = (3k - 1)$ .

$$(3k - 1)^2 - 13(3k - 1) + 36$$

$$= a^2 - 13a + 36$$

$$= (a - 9)(a - 4)$$

$$= ((3k - 1) - 9)((3k - 1) - 4)$$

$$= (3k - 10)(3k - 5)$$

Now that we've factored, we put  $(3k - 1)$  back in.

Simplify inside the brackets.

d) Factor  $x^2 - 13xy + 36y^2$

This has an extra variable, but we can use the same rule. **M:**  $36y^2$  **A:**  $-13y$  **N:**  $-9y, -4y$

$$= (x - 9y)(x - 4y)$$

Example 2: Factoring by Grouping.

Factoring by grouping is a jump start on decomposition – we have an even number of terms (usually 4) and we common factor the first pair and last pair.

$$\begin{aligned} \text{a) } & 2x^2 - 4x - 7x + 14 \\ & = 2x(x - 2) - 7(x - 2) \\ & = (x - 2)(2x - 7) \end{aligned}$$

$$\begin{aligned} \text{b) } & ab - 2b + 3ac - 6c \\ & = b(a - 2) + 3c(a - 2) \\ & = (a - 2)(b + 3c) \end{aligned}$$

$$\begin{aligned} \text{c) } & x^3 - 5x^2 - 9x + 45 \\ & = x^2(x - 5) - 9(x - 5) \\ & = (x - 5)(x^2 - 9) \\ & = (x - 5)(x - 3)(x + 3) \end{aligned} \quad \text{Difference of squares!}$$

$$\begin{aligned} \text{d) } & x^3 - 5x^2 - 8x - 48 \\ & = x^2(x - 5) - 8(x + 6) \end{aligned}$$

The brackets do not match, so this **cannot be factored** using this method.

Example 3: Combining Strategies

Factor:  $d^2 - 4d + 4 - f^2 - 12f - 36$

We have two different trinomials here, so let's work with one variable at a time.

$$\begin{aligned} & d^2 - 4d + 4 - f^2 - 12f - 36 \\ & = (d - 2)^2 - f^2 - 12f - 36 \\ & = (d - 2)^2 - (f^2 + 12f + 36) \\ & = (d - 2)^2 - (f^2 + 12f + 36) \\ & = (d - 2)^2 - (f + 6)^2 \\ & = [(d - 2) - (f + 6)][(d - 2) + (f + 6)] \\ & = [d - f - 2 - 6][d + f - 2 + 6] \\ & = [d - f - 8][d + f + 4] \end{aligned}$$

This is a perfect square trinomial.

Common factor out -1.

This is also a perfect square trinomial.

This is a difference of squares!

Simplify the brackets.

Whew!

Practice: pg. 236 #6f, 7c, 8f, 9, 10, 14\*, 17\*