

Problem Solving with Quadratics

Example 1: The “Quadratics Cup” is a new coffee shop that sells various coffee-related items, but with a quadratics twist (like an equation written on your cup, or your caramel drizzle in the shape of parabolas, etc). The profit of this new coffee shop can be described by the equation

$$P = -4n^2 + 64n - 112$$

where P represents profit in hundreds of dollars and n represents the number of customers, in thousands.

- a. How much profit is made when 3500 customers are served in a day?

Since n is in thousands, $n = 3.5$.

$$P = -4(3.5)^2 + 64(3.5) - 112$$
$$P = 63$$

Since P is in hundreds of dollars, the profit is **\$6300**.

- b. How many customers must be served for the company to break-even?

The break-even points are the **zeroes**.

$$P = -4n^2 + 64n - 112$$
$$P = -4(n^2 - 16n + 28)$$
$$P = -4(n-2)(n-14)$$

The zeroes are 2 and 14, so at 2000 and 14000 customers the company breaks even.

- c. At what number of customers does the company make the highest profit?

The maximum profit occurs at the vertex. Halfway between the zeroes is 8, so at 8000 customers the maximum profit is made.

Example 2: pg. 287

Example 3: Mr. Kempe & Mr. Varsava are developing a Math app to help students learn their math facts. After doing some research, they estimate that if they sell the app for \$2.00, 10000 people will buy it. Every time the price is dropped by \$0.05, another 100 people will buy the app. What price should they set to make the most revenue?



Let x be the number times the price is lowered.

To calculate revenue: revenue = (number sold)(price). [For example, if they keep the price of \$2, they will take in $10000 \times 2 = \$20000$.]

$$R = (2 - 0.05x)(10000 + 100x)$$

This is already in a factored form, so let's find the vertex:

$$\begin{array}{ll} 2 - 0.05x = 0 & 10000 + 100x = 0 \\ 2 = 0.05x & 10000 = -100x \\ 40 = x & -100 = x \end{array}$$

$$\text{Axis: } \frac{40 + (-100)}{2} = -30$$

$$\begin{aligned} R &= (2 - 0.05(-30))(10000 + 100(-30)) \\ R &= (3.5)(7000) \\ R &= 24500 \end{aligned}$$

Vertex is $(-30, 24500)$.

Recall that x is the number of times the price is *lowered*, not the price itself. The price is given by the expression $(2 - 0.05x)$, so from above the best price is \$3.50 for a maximum revenue of \$24500.

Practice: pg. 293 #3, 11, 14, 15, 16