MPM 2D
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Class Notes
Unit 4, Lesson 8

Vertex Form By Completing the Square

So far, we have obtained the vertex form by:

- reading it off a graph
- using factored form to find the zeroes, then the axis of symmetry, then the vertex
- using partially factored form to find "zeroes", the axis, the vertex.

In this lesson we will look at an algebraic way of doing the same thing. This process is called **completing the square**.

Recall:
A **perfect square trinomial** has the form $(a+b)^2=a^2+2ab+b^2$ [or $(a-b)^2=a^2-2ab+b^2$].
The **vertex form** of a parabola is $y=a(x-h)^2+k$.

Notice how the $(x-h)^2$ portion of the vertex is the **factored form of a perfect square**. The goal of completing the square is to create a perfect square trinomial within our expression to get vertex form. Here are the steps!

Example:	$\boldsymbol{y=2x^2+12x-10}$
Common factor *a* from the first two terms.	$y=2(x^2+6x)-10$
Determine a constant that will complete the square. To do this, take the coefficient of the *x* term, divide it in half, and square it.	$\frac{6}{2}=3$; $3^2=9$
Add and subtract this value inside the brackets.	$y=2(x^2+6x+9-9)-10$
Distribute the value of *a* to the last term in the brackets, to push it out of the brackets.	$y=2(x^2+6x+9)-18-10$
Factor the perfect square trinomial.	$y=2(x+3)^2-18-10$
Simplify the constants. Bam! We have vertex form.	$y=2(x+3)^2-28$

Example 2: Determine the vertex form of $y = x^2 - 4x - 5$.

$y=(x^2-4x)-5$
$y=(x^2-4x+4-4)-5$ [4/2 = 2; $2^2 = 4$]
$y=(x^2-4x+4)-4-5$
$y=(x-2)^2-4-5$
$y=(x-2)^2-9$

Example 3: Determine the vertex form of $y=-3x^2+2x+10$.

$y=-3x^2+2x+10$

We must factor out *a* even if it doesn't work "nicely". To force out a factor of -3 from 2, we divide, giving $-\frac{2}{3}$.

$y=-3\left(x^2-\frac{2}{3}x\right)+10$

$-\frac{2}{3}\div 2=-\frac{1}{3}$; $\left(-\frac{1}{3}\right)^2=\frac{1}{9}$

$y=-3\left(x^2-\frac{2}{3}x+\frac{1}{9}\right)+\frac{1}{3}+10$

$-3\times-\frac{1}{9}=\frac{1}{3}$

$y=-3\left(x-\frac{1}{3}\right)^2+\frac{1}{3}+\frac{30}{3}$

$y=-3\left(x-\frac{1}{3}\right)^2+\frac{31}{3}$

Practice: pg. 331 #1, 6, 7, 8, 11