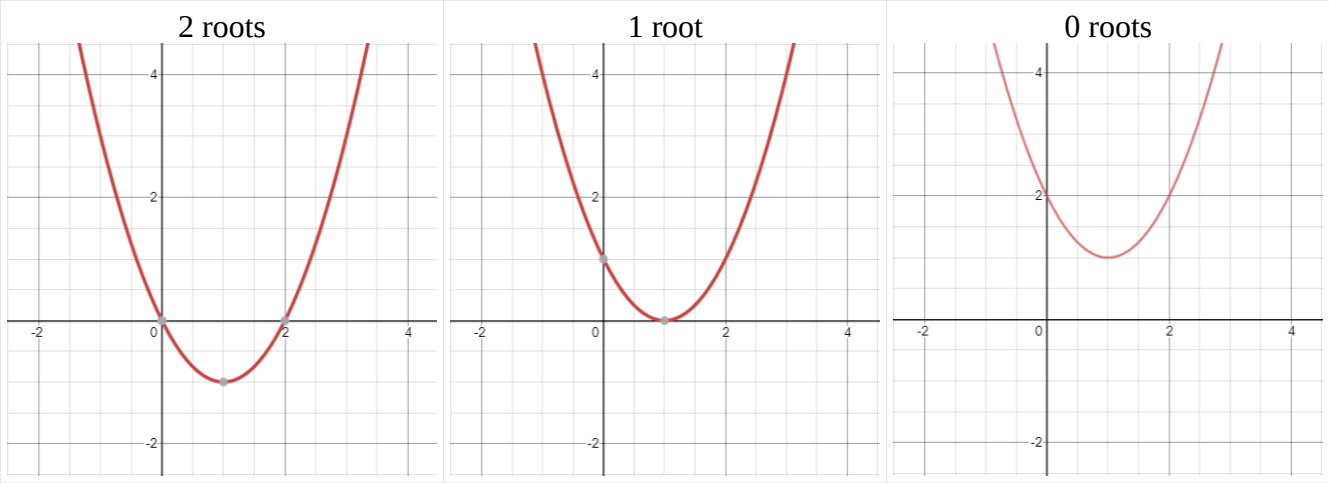
MPM 2D
Mr. Kempe

Class Notes
Unit 4, Lesson 10

# Interpreting the Roots of Quadratic Equations

Recall: How many x-intercepts (a.k.a. zeroes, roots) can a quadratic relation have?

There are three cases:




Example 1: Determine the roots of $x^2+2x+3=0$

Attempt to factor: M: 3, A: 2, N: ?? doesn't work!

Quadratic Formula:

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-2\pm\sqrt{(2)^2-4(1)(3)}}{2(1)}$$

$$x=\frac{-2\pm\sqrt{4-12}}{2}$$

$$x=\frac{-2\pm\sqrt{-8}}{2}$$

… and we're stuck, because you cannot take the square root of a negative number. Since we were unable to solve for the roots, this must have been the case of **no roots**. It would be nice to have known that before we started!

Definition: The **discriminant** of the quadratic formula is $b^2 - 4ac$. (The part under the radical sign.)

If $b^2 - 4ac > 0$, there are two (real) roots.
If $b^2 - 4ac = 0$, there is one* (real) root. (*Technically, it has one *repeated* root, or "the same root twice".)
If $b^2 - 4ac < 0$, there are no (real) roots.

Example 2: Determine the number of roots for each quadratic relation.

| a) $y = 4x^2 - 24x + 36$ | b) $y = -x^2 - 5x + 7$ | c) $y = 6x^2 + 2x + 5$ |
|---|---|---|
| $b^2 - 4ac$<br>$= (-24)^2 - 4(4)(36)$<br>$= 576 - 576$<br>$= 0$<br><br>This relation has one root. | $b^2 - 4ac$<br>$= (-5)^2 - 4(-1)(7)$<br>$= 25 + 28$<br>$= 53$<br><br>This relation has two roots. | $b^2 - 4ac$<br>$= (2)^2 - 4(6)(5)$<br>$= 4 - 120$<br>$= -116$<br><br>This relation has no roots. |

Example 3: The height, in feet, of a football kicked into the air is given by the equation $h = -18t^2 + 24t$. Can this kick clear a field goal post whose bar sits 10ft above the ground?

$10 = -18t^2 + 24t$
$0 = -18t^2 + 24t - 10$

Check the discriminant:

$$b^2 - 4ac$$
$$= (24)^2 - 4(-18)(-10)$$
$$= 576 - 720$$
$$= -144$$

Since this discriminant is negative, this equation has no solution, so the football does not go high enough!

Practice: pg. 350 #3, 6, 8, 9