

## Problem Solving Part 3

Example 1: Which equation has a lower vertex?

A:  $y = -2(x - 5)(x + 1)$

B:  $y = -3x^2 + 11x + 8$

<p>Vertex of A:</p> <p>This is already factored, so let's use the zeroes to locate the vertex.</p> <p>Zeroes are 5, -1</p> <p>Axis: <math>\frac{5 + (-1)}{2} = \frac{4}{2} = 2</math></p> <p><math>y = -2(2 - 5)(2 + 1)</math>  <math>y = -2(-3)(3)</math>  <math>y = 18</math></p> <p>The vertex is (2, 18).</p>	<p>Vertex of B:</p> <p style="text-align: center;"><math>y = -3x^2 + 11x + 8</math></p> <p style="text-align: center;"><math>y = -3\left(x^2 - \frac{11}{3}x\right) + 8</math></p> <p style="text-align: center;"><math>y = -3\left(x^2 - \frac{11}{3}x + \frac{121}{36} - \frac{121}{36}\right) + 8</math></p> <p style="text-align: center;"><math>y = -3\left(x - \frac{11}{6}\right)^2 + \frac{121}{12} + 8</math></p> <p style="text-align: center;"><math>y = -3\left(x - \frac{11}{6}\right)^2 + 18\frac{1}{12}</math></p> <p>The vertex is <math>\left(\frac{11}{6}, 18\frac{1}{12}\right)</math></p>
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Vertex A is lower (by a tiny, tiny bit!)

Example 2: The safe stopping distance, in meters, for a boat traveling at  $v$  km/h in calm water can be modeled by the relation  $d = 0.004v^2 + 0.02v + 6$ .

A boat is 100m away from shore.

- a. If the boat is traveling at 20km/h, can it stop safely?

$$d = 0.04(20)^2 + 0.02(20) + 6$$

$$d = 16 + 0.4 + 6$$

$$d = 22.4$$

The boat only needs 22.4m of stopping distance, so yes, it can stop safely.

- b. What is the fastest speed a boat could be traveling at to stop safely?

$$100 = 0.04v^2 + 0.02v + 6$$

$$0 = 0.04v^2 + 0.02v - 94$$

$$d = \frac{-0.02 \pm \sqrt{0.02^2 - 4(0.04)(-94)}}{2(0.04)}$$

$$d \approx 46.7, -47.2$$

The fastest speed is about 46 km/h.

Example 3: Determine the point(s) of intersection of the line  $y = 2x - 5$  and  $y = 0.5(x - 1)^2 - 1$ .

Substitution!

$$2x - 5 = \frac{1}{2}(x - 1)^2 - 1$$

$$4x - 10 = (x - 1)^2 - 2$$

$$4x - 10 = x^2 - 2x + 1 - 2$$

$$0 = x^2 - 2x - 4x + 1 - 2 + 10$$

$$0 = x^2 - 6x + 9$$

$$0 = (x - 3)^2$$

Therefore  $x = 3$ , and the (single) point of intersection is  $(3, 1)$ .