

Test #4 – Quadratics Part 2

[40 marks]

Part A: Multiple Choice [K/U, 10 marks]

Questions 1 – 5 use the quadratic relation $y=2(x+3)^2$.

- What is the **vertex** of this relation?
a. (2,3) b. (2, -3) **c.** (-3, 0) d. (3, 0)
- What **direction** does this relation open?
a. Up b. Down c. Left d. Right
- What is the **step-pattern** of this relation?
a. 1, 3, 5 **b.** 2, 6, 10 c. 3, 9, 15 d. -3, -9, -15
- How many **zeroes** does this relation have?
a. 0 **b.** 1 c. 2 d. 3
- What is the **y-intercept** of this relation?
a. 0 b. 6 **c.** 18 d. 36
- What are the solution(s) to the equation $2x^2+8=0$?
a. $x = 2$ b. $x = -2$ c. $x = \pm 2$ **d.** No solutions
- How many** solutions are there to the equation $y=3x^2-8x+2$?
a. 0 b. 1 **c.** 2 d. 3
- What are the solution(s) to the equation $(x-3)(2x+1)=0$?
a. 3, -1 b. -3, 1 **c.** 3, $-\frac{1}{2}$ d. -3, $\frac{1}{2}$
- The height of a football thrown into the air is given by the equation $h=-5t^2+10t+2$, where h is the height in meters and t is the time in seconds. If you were asked to find the maximum height of the ball, you would need to find the...
a. y-intercept b. Zeroes **c.** Vertex d. Step Pattern
- The profit function for a coffee shop is given by the equation $P=-2n^2+12n-42$, where P is the profit in thousands of dollars, and n the number of customers, also in thousands. If you were asked to find the break-even point for this coffee shop, you would need to find the...
a. y-intercept **b.** Zeroes c. Vertex d. Axis of Symmetry

Part B: Definition / Short Answer

1. The vertex form is $y = a(x - h)^2 + k$. Describe the role of a , h , and k in determining the shape/position of a parabola. [6 marks]

a :

Controls the direction of opening (up if $a > 0$; down if $a < 0$).
Affects the step pattern (stretch): $1a, 3a, 5a$.

h :

Horizontal translation. Moves the parabola right (if $h > 0$) or left (if $h < 0$).

k :

Vertical translation. Moves the parabola up (if $k > 0$) or down (if $k < 0$).

2. Describe two ways you can get to **vertex form** from **standard form**. [2 marks]

- i) Factor (or partially factor), find the roots, then axis of symmetry, then vertex.
- ii) Complete the square!

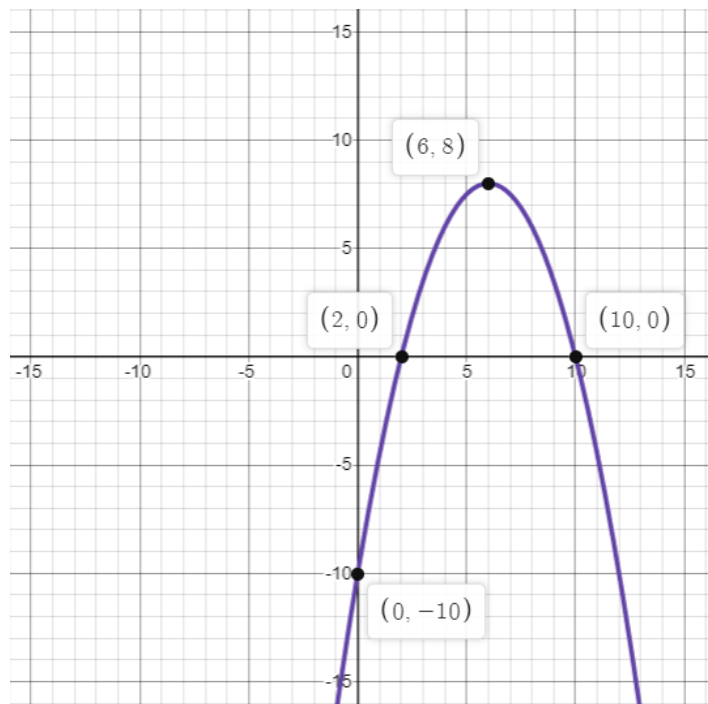
3. Describe how the **discriminant** is used to determine the number of roots of a quadratic relation. [2 marks]

The discriminant is $b^2 - 4ac$ (the part under the root sign of the quadratic formula).
If this value is positive, there are two roots.
If this value is negative, there are no roots.
If this value is 0, there is one root.

Part C: Problem Solving [ATIPS, 20 marks]

Complete any 5 of the following 6 problems. Each problem worth is 4 marks.

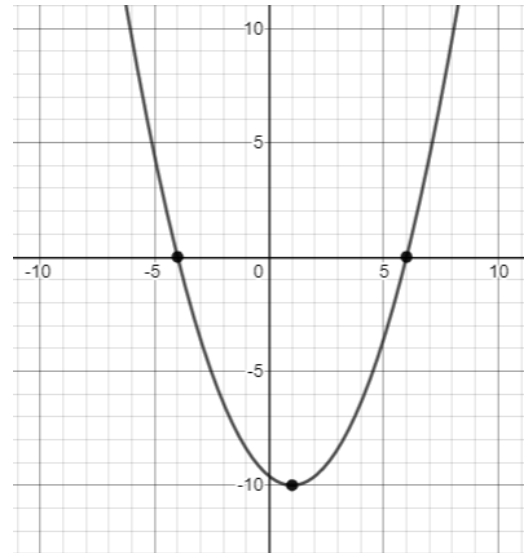
1. Sketch the graph of $y = -\frac{1}{2}(x-6)+8$ on the grid provided.
Label the key features of the parabola.



2. Determine the equation of the parabola shown in **vertex form**.

The vertex is at (1, -10). Another point on the parabola is (6, 0).

$$\begin{aligned}y &= a(x-h)^2 + k \\y &= a(x-1)^2 - 10 \\0 &= a(6-1)^2 - 10 \\10 &= a(25) \\ \frac{10}{25} &= a \\ \frac{2}{5} &= a \\ y &= \frac{2}{5}(x-1)^2 - 10\end{aligned}$$



3. a. Convert $y=5x^2-80x+320$ to vertex form.

$$\begin{aligned}y &= 5x^2 - 80x + 320 \\y &= 5(x^2 - 16x) + 320 \\y &= 5(x^2 - 16x + 64 - 64) + 320 \\y &= 5(x^2 - 16x + 64) - 320 + 320 \\y &= 5(x - 8)^2\end{aligned}$$

- b. Verify your work by converting your answer from (a) back to standard form.

$$\begin{aligned}y &= 5(x - 8)^2 \\y &= 5(x - 8)(x - 8) \\y &= 5(x^2 - 16x + 64) \\y &= 5x^2 - 80x + 320\end{aligned}$$

4. Determine the roots of $y = 3x^2 - 17x + 2$. Round your final answer to 2 decimal places.

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$y = \frac{17 \pm \sqrt{(-17)^2 - 4(3)(2)}}{2(3)}$$

$$y = \frac{17 \pm \sqrt{289 - 24}}{6}$$

$$y = \frac{17 \pm \sqrt{265}}{6}$$

$$y \approx 5.55, 0.12$$

5. Tickets to a school dance cost \$5, and the projected attendance is 300 people. For every \$0.50 increase in the ticket price, the dance committee projects that attendance will decrease by 20. What ticket price will generate \$1562.50 in revenue?

Revenue = (cost)(number sold)

$$\begin{aligned}R &= (5 + 0.5n)(300 - 20n) \\R &= 1500 - 100n + 150n - 10n^2 \\R &= 1500 + 50n - 10n^2\end{aligned}$$

$$\begin{aligned}1562.50 &= 10n^2 + 50n + 1500 \\0 &= -10n^2 + 50n - 62.50\end{aligned}$$

$$\begin{aligned}y &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\y &= \frac{-50 \pm \sqrt{(50)^2 - 4(-10)(-62.5)}}{2(-10)} \\y &= \frac{-50 \pm \sqrt{0}}{-20} \\y &= \frac{-50}{-20} \\y &= \frac{5}{2}\end{aligned}$$

The ticket price is $5 + 0.5\left(\frac{5}{2}\right) = \6.25 .

6. Skydivers jump out of an airplane at an altitude of 3.5 km. The equation $H = 3500 - 5t^2$ models the altitude, H , in metres, of the skydivers at t seconds after jumping out of the airplane.
- a. How far have the skydivers fallen after 10 s?

$$\begin{aligned}H &= 3500 - 5t^2 \\H &= 3500 - 5(10)^2 \\H &= 3500 - 500 \\H &= 3000\end{aligned}$$

Since the initial height was 3500, they have dropped $3500 - 3000 = \mathbf{500m}$.

- b. The skydivers open their parachutes at an altitude of 1000 m. How long did they free fall?

$$\begin{aligned}1000 &= 3500 - 5t^2 \\-2500 &= -5t^2 \\ \frac{-2500}{-5} &= t^2 \\ 500 &= t^2 \\ \sqrt{500} &= t \\ t &\approx 22.36\end{aligned}$$

They were in free fall for about 22 seconds.