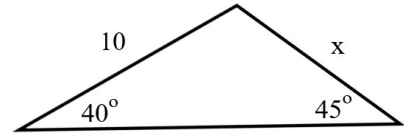


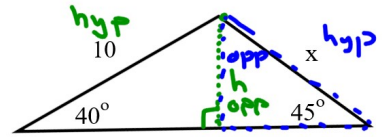
# Sine Law

Example 1: Determine the length of  $x$ .

This is not a right-angled triangle, so we cannot use the primary trig ratios directly. However, there's nothing to stop us from splitting the triangle into *two* right triangles by drawing in the height of the triangle. (Note this problem is now similar to the last example from the previous lesson.)



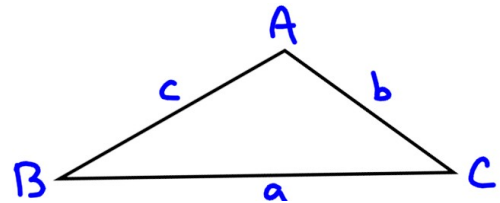
$$\begin{aligned} \sin 40^\circ &= \frac{h}{10} & \sin 45^\circ &= \frac{6.43}{x} \\ 10 \sin 40^\circ &= h & x &= \frac{6.43}{\sin 45^\circ} \\ 6.43 &\approx h & x &\approx 9.09 \end{aligned}$$



Because we can do this with *any* triangle, we can develop a new rule based on this idea.

## Labeling Triangles

We label the vertices of the triangle with capital letters. We label the side *across* from the vertex with the same letter, but lowercase.



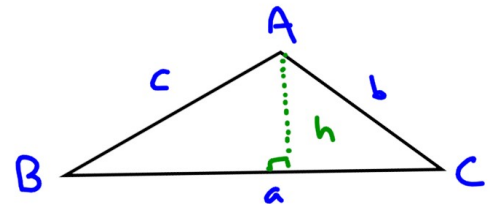
## Derivation of Sine Law

Drawing in our height, we note that in the triangle on the *left*:

$$\sin B = \frac{h}{c} \quad \text{and, solving for } h, \text{ we get } h = c \sin B$$

and on the *right*:

$$\sin C = \frac{h}{b} \quad \text{and, solving for } h, \text{ we get } h = b \sin C$$



This means that  $c \sin B = b \sin C$ . Dividing both sides by  $b$  and  $c$  gives us:

$$\frac{\sin B}{b} = \frac{\sin C}{c} \quad \text{which is known as the **Sine Law** .}$$

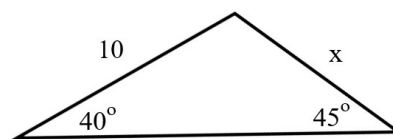
## Sine Law

In any triangle,  $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$  (also  $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$  ).

[Both versions always work; personally, I prefer to place my unknown value in the numerator.]

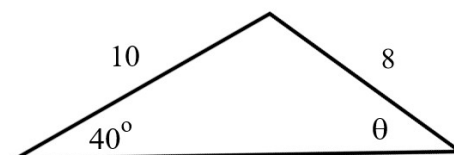
Example 2: Repeat Example 1 using the Sine Law.

$$\frac{x}{\sin 40} = \frac{10}{\sin 45}$$
$$x = \frac{10 \sin 40}{\sin 45}$$
$$x = \frac{10(0.64278760 \dots)}{(0.70710678 \dots)}$$
$$x \approx 9.09$$



Example 3: Determine the measure of angle  $\theta$ .

$$\frac{\sin \theta}{10} = \frac{\sin 40}{8}$$
$$\sin \theta = \frac{10 \sin 40}{8}$$
$$\sin \theta = 0.80348451 \dots$$
$$\theta = \sin^{-1} 0.80348451$$
$$\theta \approx 53.5$$



Practice: pg. 433 #3, 4, 7, 11