

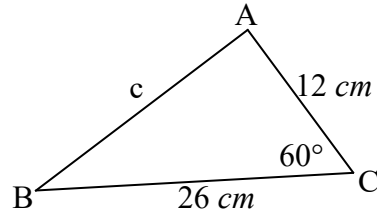
Cosine Law

(Sides)

Example 1: Solve for c .

We attempt the sine law

$$\frac{26}{\sin A} = \frac{c}{\sin 60^\circ} \quad \text{or} \quad \frac{12}{\sin B} = \frac{c}{\sin 60^\circ}$$



Unfortunately, there is not enough information to solve for c using the sine law. We cannot even solve for angles A or B! This calls for a new tool: the cosine law!

The Cosine Law is:

$$c^2 = a^2 + b^2 - 2ab \cos C$$

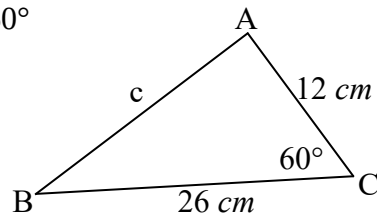
Notice the similarities to Pythagorean Theorem... we are essentially “adjusting” it because angle C is not a right angle.

The cosine law is written in different ways, depending which angle is involved.

$$\begin{aligned} c^2 &= a^2 + b^2 - 2ab \cos C \\ \text{or } b^2 &= a^2 + c^2 - 2ac \cos B \\ \text{or } a^2 &= b^2 + c^2 - 2bc \cos A \end{aligned}$$

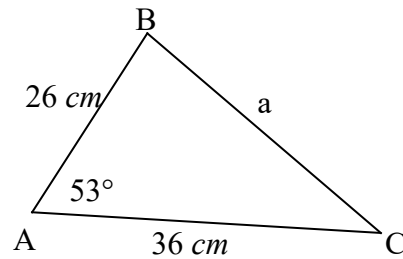
Example 1: Now we can solve for c from our first problem (*redrawn below*)

$$\begin{aligned} c^2 &= a^2 + b^2 - 2ab \cdot \cos C \\ c^2 &= (26)^2 + (12)^2 - 2(26)(12) \cdot \cos 60^\circ \\ c^2 &= 676 + 144 - 624 \cdot (0.5) \\ c^2 &= 676 + 144 - 312 \\ c^2 &= 508 \\ \sqrt{c^2} &= \sqrt{508} \\ \therefore c &= 22.54 \text{ cm} \end{aligned}$$



Example 2: Solve for a .

$$\begin{aligned}a^2 &= b^2 + c^2 - 2bc \cos A \\a^2 &= 36^2 + 26^2 - 2(36)(26) \cos 53^\circ \\a^2 &= 1296 + 676 - 563.2848 \\a^2 &= 845.4304 \\a &= \sqrt{845.4304} \\a &\approx 29.1\end{aligned}$$

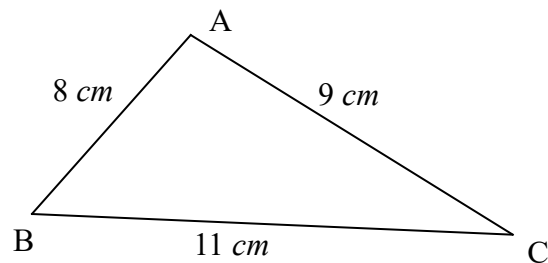


(Angles)

Example 3: Determine the measure of angle A.

We cannot use the sine law because we do not know any angles. We turn to the cosine law involving angle A:

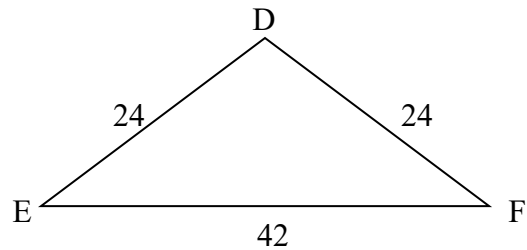
$$a^2 = b^2 + c^2 - 2bc \cos A$$



Substituting the sides, we will work the equation to find $\cos A$, and then A.

$$\begin{aligned}11^2 &= 9^2 + 8^2 - 2(9)(8) \cos A \\121 &= 81 + 64 - 144 \cos A \\121 &= 145 - 144 \cos A \\121 - 145 &= -144 \cos A \\-24 &= -144 \cos A \\\frac{-24}{-144} &= \cos A \\0.16666667 &= \cos A \\A &= \cos^{-1}(0.16666667) \\A &= 80.4^\circ\end{aligned}$$

Example 4: Solve triangle DEF.



$$\begin{aligned}d^2 &= e^2 + f^2 - 2ef \cos D \\42^2 &= 24^2 + 24^2 - 2(24)(24) \cos D \\1764 &= 576 + 576 - 1152 \cos D \\1764 &= 1152 - 1152 \cos D \\1764 - 1152 &= -1152 \cos D \\612 &= -1152 \cos D \\\frac{612}{-1152} &= \cos D \\-0.53125 &= \cos D \\D &= \cos^{-1}(-0.53125) \\D &= 122\end{aligned}$$

$$\begin{aligned}e^2 &= d^2 + f^2 - 2df \cos E \\24^2 &= 42^2 + 24^2 - 2(42)(24) \cos E \\576 &= 1764 + 576 - 2016 \cos E \\576 &= 2340 - 2016 \cos E \\576 - 2340 &= -2016 \cos E \\-1764 &= -2016 \cos E \\\frac{-1764}{-2016} &= \cos E \\0.875 &= \cos E \\E &= \cos^{-1}(0.875) \\E &= 29\end{aligned}$$

Similarly to E , angle F is also 29°

Practice: pg. 444 # 3 - 6